

# Synthetic Aperture Ultrasonic Imaging Using Phase Shift Migration Algorithm

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## Abstract

Ultrasonic testing is an important method in nondestructive testing industry. Ultrasonic image processing helps workers to understand the location, size and shape of defects faster and better. Phase shift migration algorithm is a new method of ultrasonic imaging algorithm. In this paper, after collecting ultrasonic data and using adaptive filter, the ultrasonic image with higher resolution is obtained by phase shift migration algorithm.

## Keywords

Adaptive filter, Ultrasonic imaging, PSM algorithm.

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## 1. Introduction

Phase shift migration (PSM) is an ultrasonic imaging method that combines the migration technique in reflection seismology with SAFT in frequency domain. It is an image reconstruction method based on frequency domain. Compared with SAFT technology in time domain, it has higher range resolution, higher azimuth resolution and lower side lobe, and its image quality is higher. The technology is derived from seismic imaging and synthetic aperture radar. SAFT is a migration technique in seismic imaging. Computer-based seismic data processing method originated in the 1970s and were limited to the time-space domain initially.

## 2. Theory

### 2.1 PSM algorithm

Phase shift migration was derived from wave equations.  $x$  is the scanning axis of the transducer,  $z$  is the depth direction,  $t$  is the time, and  $c$  is the wave velocity in the medium,  $p(x, z, t)$  denote the wave field generated by a set of exploding reflects. The wave equation is given by:

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] p(x, z, t) = 0 \quad (1)$$

Taking a three-dimensional Fourier transform over the coordinates yield

$$\left[ k_x^2 + k_z^2 - \frac{\omega^2}{c^2} \right] P(k_x, k_z, \omega) = 0 \quad (2)$$

Where  $k_x$  and  $k_z$  are the x- and z-direction components of the wave-number vector, respectively,  $\omega$  is the angular frequency. Since  $P(k_x, k_z, \omega) \neq 0$ , it must satisfy the dispersion relation of 2-D wave equation:

$$k_x^2 + k_z^2 = \frac{\omega^2}{c^2} \quad (3)$$

Then take a 2-D Fourier transform of (1) over  $x$  and  $t$ , we get

$$\left[ k_x^2 - \frac{\partial^2}{\partial z^2} - \frac{\omega^2}{c^2} \right] P(k_x, z, \omega) = 0 \quad (4)$$

Inserting (3) into (4), we can obtain the expression:

$$\left[ k_z^2 + \frac{\partial^2}{\partial z^2} \right] P(k_x, z, \omega) = 0 \quad (5)$$

In ultrasonic testing applications, we only consider the up-going waves (negative z-axis) and negative  $\omega$ . Hence, (5) is reduced to

$$\frac{\partial}{\partial z} P(k_x, z, \omega) = -ik_z P(k_x, z, \omega) \quad (6)$$

The solution to (6) has the form:

$$P(k_x, z, \omega) = P(k_x, z_0 = 0, \omega) e^{-ik_z z} \quad (7)$$

where  $z_0$  is the minimum of  $z$ . Inserting (3) into (7), for  $\omega < 0$ , we can obtain

$$P(k_x, z, \omega) = P(k_x, z_0 = 0, \omega) \alpha(\omega, k_x, z, c) \quad (8)$$

where

$$\alpha(\omega, k_x, z, c) = \exp \left\{ i \sqrt{\frac{\omega^2}{c^2} - k_x^2} z \right\} \quad (9)$$

and for  $\omega > 0$  we have  $P(k_x, z, \omega) = 0$ .

Taking the inverse Fourier transform of  $P(k_x, z, \omega)$  with respect to  $k_x$  and  $\omega$ , when  $t=0$ , we can get  $p(x, z, t)$  which is the high-resolution image of the depth direction(z-axis):

$$p(x, z = 0, t) = \frac{1}{4\pi^2} \iint P(k_x, z, \omega) e^{ik_x x} e^{i\omega t} d\omega dk_x. \quad (10)$$

for  $t=0$ ,  $e^{i\omega t} = 1$ . We can take the integral of  $P(k_x, z, \omega)$  over  $\omega$  and then evaluate an inverse 1D Fourier transform over  $k_x$  to reduce the amount of computation.

### 3. Experiment

The experiment was performed to verify the imaging performance of PSM algorithm. An aluminium block with side-drilled holes is placed. The block is 80mm thick which is given in Figure 1. A B-scan of the arrangement was performed using 5MHz transducer with 14mm diameter, moved in steps of 1 mm, with a sampling frequency of 25MHz.

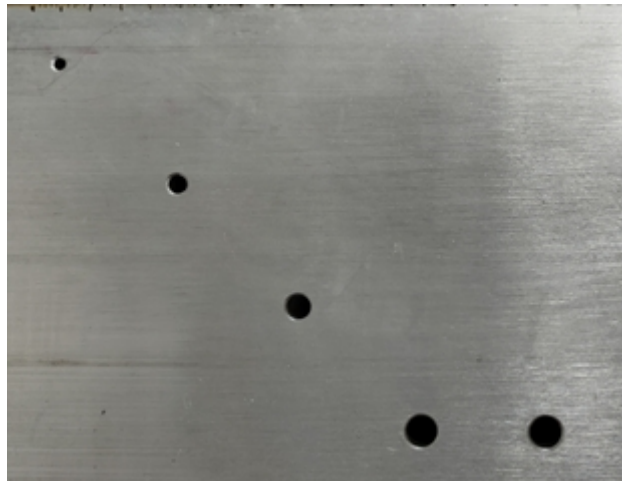


Figure 1. Aluminium block

#### 3.1 Experimental data preprocessing

The original data obtained from the scanning contains not only the information of the defect points, but also the noise signal. The noise signal includes the structural noise of the system, the echo of the test block surface and bottom, and the random noise. Since, before the algorithm is processed, we need to filter the original data.

Adaptive filtering uses different criteria to determine the adaptive weight and uses different adaptive algorithms to minimize the cost function. The results of adaptive filter denoised given by Figure 2.

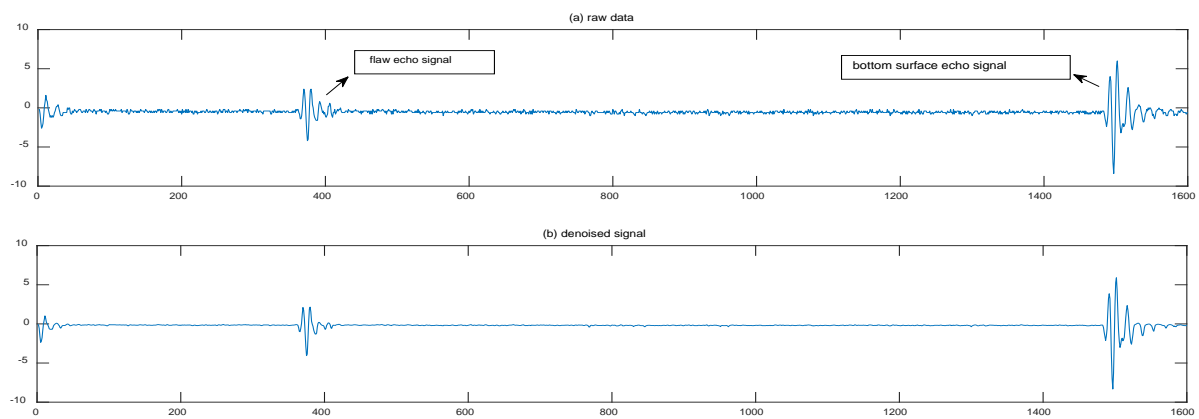


Figure 2. One frame of ultrasonic echo is denoised. The adaptive filter has a good filtering effect.

### 3.2 Results after focusing

In this paper, the -6dB method is used to estimate the resolution of image. The theory of this method is to take the length of abscissa which is 6dB lower than the maximum echo amplitude as the standard.

Figure 3 is the original waveform of scanning the target defect area. In order to evaluate the focusing effect, we take the single flaw of the raw image and the focused image.

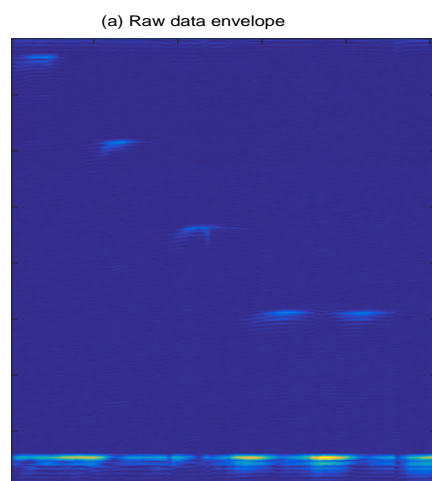


Figure 3. Raw data and PSM focused image from experiment

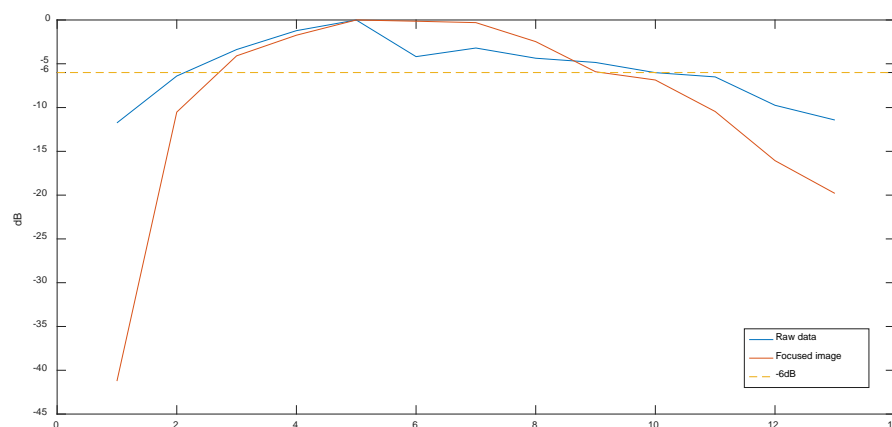


Figure 4. -6dB width of flaws

Figure 4 shows that the -6db width of the focused image is less than the width of the raw image. We choose 13 sampling points. After processing, -6db width of raw image which is 9 sampling points is changed into 7 sampling points and precision is improved nearly 22%.

#### 4. Conclusion

This paper introduces the theory of PSM algorithm in detail. Using the existing experimental instruments, a straight probe is used to scan the different defects on the standard test block, and after obtaining the raw data, adaptive filtering is carried out to process it, and then data is processed by the above algorithm. Finally, the decibels drop method is used to estimate the resolution and the result showed that the resolution improved nearly 22%. However, it is found from the experimental results that the experimental resolution of this paper does not reach a satisfactory result.

#### References

- [1] Skjeltvareid M H , Olofsson T , Birkelund Y , et al. Synthetic aperture focusing of ultrasonic data from multilayered media using an omega-K algorithm[J]. IEEE Transactions on Ultrasonics Ferroelectrics & Frequency Control, 2011, 58(5):1037.
- [2] Du Yinghua. Research on synthetic aperture focusing technology for ultrasonic imaging [D]. Tianjin University, Tianjin, 2010.
- [3] Olofsson T , Stepinski T . Phase shift migration for imaging layered materials and objects immersed in water[J]. 2009.
- [4] K. Gu, G. Wang and J. Li. Migration based SAR imaging for ground penetrating radar systems [J]. IEEE Proceedings - Radar, Sonar and Navigation, vol. 151, no. 5, pp. 317-325, 10 Oct. 2004.
- [5] Yang Chun. Research on the synthetic aperture focusing technique for ultrasonic imaging of layered objects [D]. Tsinghua University, Beijing, 2014.
- [6] Information on <https://www.zhihu.com/question/22611929?sort=created>.