

Research on Reliability Modeling and Evaluation Method Based on MTBF Definition

Hongxiang Zhu^a, Zhifeng Cheng and Lu Sun

Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences.
Changchun, Jilin, 130022, China.

^a947154223@qq.com

Abstract

With the gradual improvement of the reliability level for the system, the sample size of failures is different, and the MTBF (Mean Time Between Failure) of different systems also has great differences. In view of the difference in accuracy when using different reliability modeling methods to evaluate the reliability of different MTBF systems, this paper proposes use MTBF as an index to select reliability modeling and evaluation methods. In an example, the effectiveness of the method proposed in this paper is verified by modeling and analysis of the Weibull distribution and the power-law distribution.

Keywords

Wireless sensor network; Heterogeneous ant colony.

1. Introduction

In the process of system reliability evaluation, there is no evidence to choose reliability modeling and evaluation methods, which leads to low accuracy of reliability evaluation results, and can not accurately reflect the system reliability level. Choosing the correct reliability modeling and evaluation method can more accurately conduct other technical research on system reliability. Therefore, this paper proposes a method of reliability modeling and evaluation based on MTBF definition to achieve accurate evaluation of system reliability.

When the failure time satisfies the assumption of independent and identical distribution, the two Weibull distribution [1], exponential distribution function [2] and other models are generally used to evaluate the reliability of the system using failure time data. Wang Shan uses a reliability modeling method that considers factors such as uncertainty and maintenance level to evaluate the reliability of the system [3]. When the shape parameters of the Weibull distribution function take different values, a variety of distribution functions such as the normal distribution can be fitted. Therefore, the Weibull distribution function is generally used to assess the system reliability level [4]. When the failure time satisfies the premise of non-independent and identically distributed assumptions, it is generally necessary to consider the system reliability modeling under the conditions of failure competition and failure [5] and uncertainty [6].

The system has a variety of reliability data types in the actual operation process, and it is necessary to use multi-source hierarchical information sets to fuse multiple types of data to evaluate the reliability of the system. Guo J and Wilson AG proposed a Bayesian method to evaluate the reliability of complex systems under the coexistence of subjective and objective information of multi-component systems, which can be applied to realize the reliability evaluation of CNC machine tools [7]. Johnson V E and others apply multi-layer Bayesian networks to qualitatively analyze the manufacturing quality of complex equipment, and introduce sparse and system test data to evaluate

the reliability of complex systems [8]. Wu Xiaohui et al. considered the difference in stress environment between internal and external fields, and used Bayesian estimation method to fuse internal field accelerated degradation test and external field degradation information to evaluate the reliability of complex equipment [9].

The performance of the system gradually decreases with the extension of the use time, and the probability of failure gradually increases. The failure data and performance detection data can be used to perform reliability modeling and analysis on the system. Meeker W Q etc. considered the influence of temperature on the time of failure and established a performance degradation reliability evaluation model [10].

There are many types of system reliability modeling and evaluation methods, and it is of great significance to correctly evaluate the reliability of the system. Therefore, this paper proposes a research on reliability modeling and evaluation methods based on MTBF definition, and selects high accuracy for system reliability evaluation Modeling and evaluation methods.

2. Reliability modeling

2.1 Reliability model

2.1.1 Weibull distribution model

In reliability modeling and evaluation, the most commonly used probability distribution function is the Weibull distribution function. Its failure rate function can be expressed as:

$$\lambda(t) = \frac{\nu}{\psi} \left(\frac{t}{\psi} \right)^{\nu-1} \quad (2.1)$$

While ν is the scale parameter of the Weibull distribution. ψ is the shape parameter of Weibull distribution. t is the working time TBF between failures.

The reliability function can be expressed as:

$$R(t) = \exp\left[-\int_0^t \frac{\nu}{\psi} \left(\frac{t'}{\psi} \right)^{\nu-1} dt'\right] = \exp\left(-\left(\frac{t}{\psi}\right)^\nu\right) \quad (2.2)$$

The probability density function can be expressed as:

$$f(t) = -\frac{dR(t)}{dt} = \frac{\nu}{\psi} \left(\frac{t}{\psi} \right)^{\nu-1} \exp\left(-\left(\frac{t}{\psi}\right)^\nu\right) \quad (2.3)$$

The cumulative distribution function can be expressed as:

$$F(t) = 1 - R(t) = 1 - \exp\left(-\left(\frac{t}{\psi}\right)^\nu\right) \quad (2.4)$$

When the Weibull distribution model is used to fit the failure data, the mean time between failures MTBF is:

$$MTBF = \int_0^\infty t f(t) dt = \psi \Gamma\left(\frac{1+\nu}{\nu}\right) \quad (2.5)$$

2.1.2 Power law distribution

Assuming that the working time (t) between failures of the system obeys a two-parameter power law distribution, its failure rate function $\lambda(t)$:

$$\lambda(t) = \frac{t^\beta}{\eta^{\beta+1}} \quad (2.6)$$

While β is the shape parameter of the power law distribution. η is the scale parameter of the power law distribution.

The calculation methods of the reliability function, probability density function, cumulative distribution function, and MTBF of the power law distribution are the same as formulas (2.2), (2.3), (2.4) and (2.5).

2.2 Maximum Likelihood Method

The reliability model parameters can be solved by the maximum likelihood method, a series of random observations, the working time between failures t_1, t_2, \dots, t_n , and the probability density function is $f(t, v_i)$.

$$L(v_1, v_2, \dots, v_n) = \prod_{i=1}^n f(t_i, v_1, v_2, \dots, v_n) \quad (2.7)$$

Eq.2.8 can be obtained:

$$\frac{\partial \ln L(v)}{\partial v} = 0 \quad (2.8)$$

When the maximum likelihood estimate exists or the error is within 10^{-3} , the model parameters can be obtained.

2.3 Goodness of fit test

Generally, the KS test method is used to test the fitting accuracy of the model, referred to as K-S test. The K-S test statistics can be expressed as:

$$D = \max |F_n(t) - F(t)| \quad (2.9)$$

While $F_n(t)$ is the experience accumulation function.

$$F_n(t) = (j - 0.3) / (n + 0.4). \quad (2.10)$$

With a given confidence level, when is calculated from the test data, it can be judged that the test data does not obey the distribution function. When it can be judged that the test data obey the distribution function, and the smaller the value of the K-S test statistic D , the higher the fitting accuracy of the distribution function.

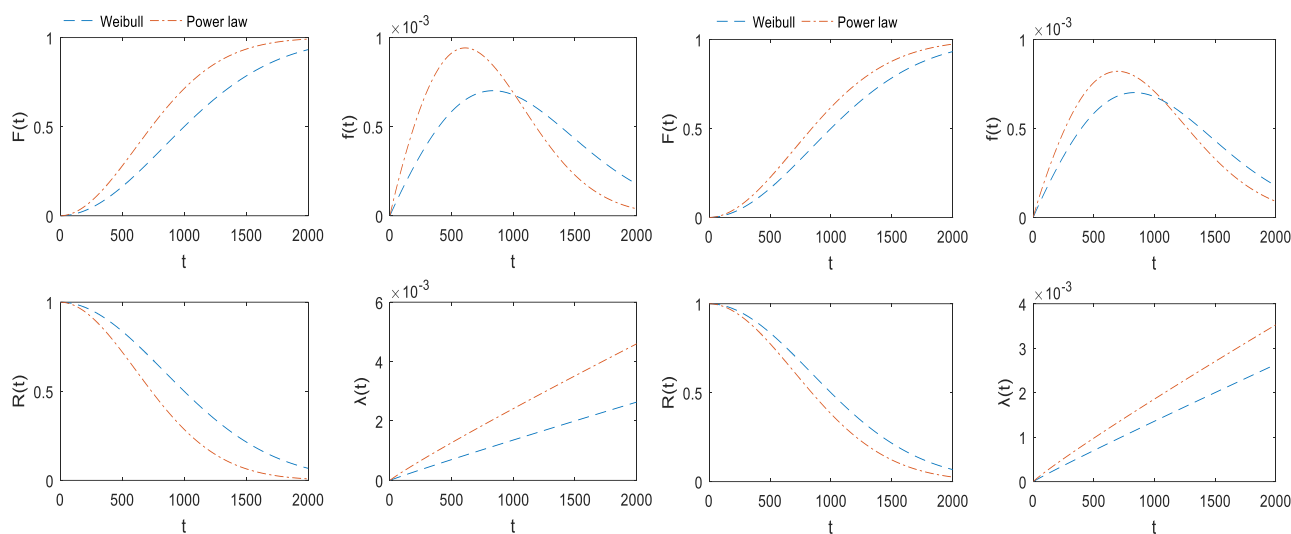
3. Case analysis

Using Weibull distribution inverse operation to randomly generate MTBF of 621.9, 820.2, 1068.4. The Weibull distribution and the power-law distribution are used to fit the data of the working time between failures in three different MTBF situations, and the maximum likelihood estimation method is used to obtain the parameters of the two different distributions as shown in Table 3.1. The failure rate, probability density function, reliability function and cumulative distribution function images of the two distribution models under different MTBF conditions are shown in Figure 3.1.

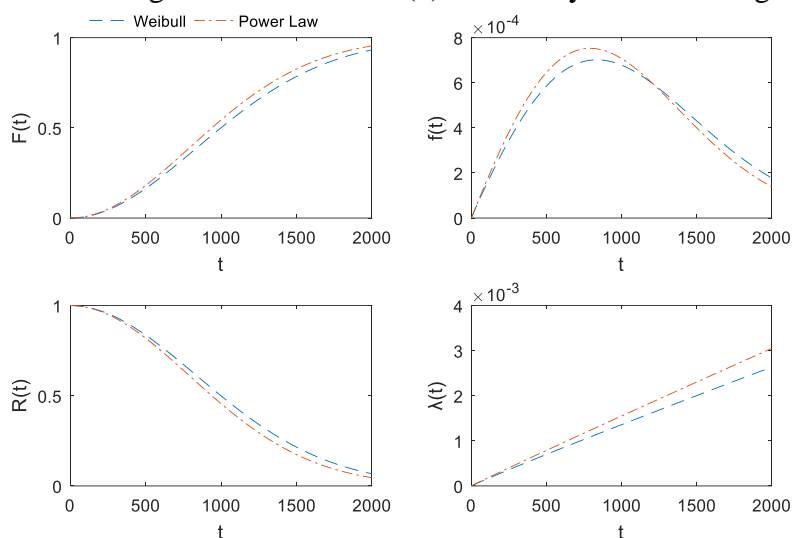
Table 3.1 Model parameter estimation

Mode	Weibull distribution			Power law distribution		
Shape parameter	2.0374	1.9543	1.9564	0.9865	0.9247	0.9756
Shape Scale parameter	702	925	1205	633	725	801
MTBF/h	621.9	820.2	1068.4	792.6	903.7	1002.2

Different distribution models are tested for the goodness of fit under different MTBFs. This paper uses the Kolmogorov-Smirnov test method to test the model fit accuracy, as shown in Table 3.2. It can be seen from Table 3.2 that when the MTBF is 621.9 and 820.2, the Weibull distribution function fitting accuracy is higher, but the difference of the KS test statistic is getting smaller. When the MTBF is 1068.4, the power law distribution fits The accuracy is better. It can be proved that when the MTBF is different, the fitting accuracy of different fitting functions is different. We can choose the appropriate reliability model according to the MTBF value.



(a) Reliability function image at MTBF=621.9 (b) Reliability function image at MTBF=820.2



(c) Reliability function image at MTBF=1068.4

Figure 3.1. Reliability function image under different MTBF conditions

Table 3.2 K-S test statistics

	D	Weibull	Power law	Difference
MTBF	621.9	0.1046	0.2274	0.1228
	820.2	0.2555	0.3213	0.0658
	1068.4	0.4307	0.3939	-0.0368

4. Conclusion

In view of the difference in accuracy when using different reliability modeling methods to evaluate the reliability of the system under different MTBF conditions, this paper carries out verification and analysis. In the example, the Weibull distribution and the power-law distribution are modeled and analyzed. When the MTBF is 621.9 and 820.2, the Weibull distribution function has a higher fitting accuracy. When the MTBF is 1068.4, the power-law distribution The fitting accuracy is good, and

the results show that when the system is at different MTBF, different reliability models should be used for evaluation.

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