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An Empirical Model of Slope Stability Based on Linear Regression

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Abstract

To obtain an empirical model of the relationship between slope stability and rainfall in Tiemagou, Bomi County, Tibet Autonomous Region, China, linear regression was used to analyze the rainfall and three-dimensional displacement data collected by a GNSS landslide displacement monitoring system in July. The results show that there is a linear relationship between the rainfall and the three-dimensional displacement and that the amount of rainfall is closely related to the slope stability. An empirical model linking the rainfall and slope stability was then developed by analyzing the data acquired by the landslide monitoring system. This model can reflect the slope stability well and is suitable for application to slopes in areas where the geological conditions are similar to those at Tianmagou.

Keywords

GNSS landslide displacement monitoring; Linear regression; Empirical model.

1. Introduction

China is a vast country with huge differences in geological conditions from east to west and north to south. The unique geological conditions in China mean that natural disasters occur widely across the country. Among these natural disasters, landslides are one of the most serious types; landslides are characterized by their high frequency of occurrence, variety of types, wide distribution, and seriousness [1]. Rainfall-induced landslides are the most common type of landslide. From an analysis of slopes in the region of the Three Gorges reservoir, Jiang Yongdong et al[2]. concluded that water is one of the most important factors affecting the stability of slopes. Following this, more models related to slope stability have been proposed. Chen Shanxiong et al[3]. proposed a slope-prediction model that depended on the amount of rainfall. Zhang Yonglei et al[4]. then established an autoregressive model using the empirical model. Based on these earlier studies, in this research, a linear regression model that can be used to evaluate and analyze the stability of slopes was developed.

The frequency of landslides in southeast Tibet is among the highest in China, and the region also experiences a variety of landslide types [1]. Tianmopou, which is located near Songhuan Village, Bomi County, Nyingchi City, Tibet Autonomous Region, lies in the middle and lower reaches of Palongzangbu river, which is oriented approximately northeast to southwest, and is surrounded by forest-covered mountains. A glacier which overhangs Tianmagou is the source of a lot of the moraines in this gully. The lower part is bare bedrock, whereas the upper part consists of a cliff wall. The gully is located at 29° 59' N, 95° 19'E and is covered by a small amount of snow all year round. Three large debris flows have occurred in this gully since 2010, and a lot of residual slope deposits remain in the gully after scouring by debris flows. The slope that has formed in the gully where the debris flows have occurred has an angle of more than 60° and is prone to landslides due to external erosion caused by, for example, rainfall. A complete GNSS landslide monitoring system was set up on an unstable slope within this gully to allow real-time monitoring of the slope. The GNSS landslide detection system and the geographical location of the study area are shown in Figure 1 and Figure 2, respectively.

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Figure 1 The GNSS monitoring system at the Tianmogou gully

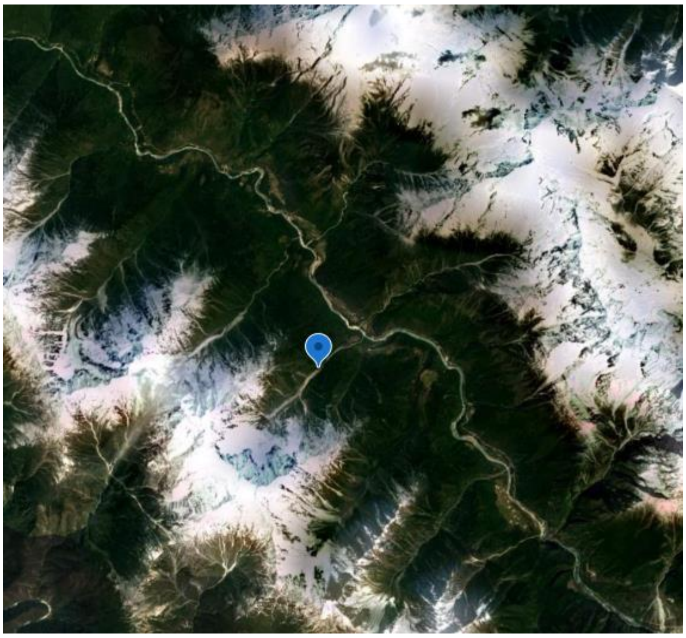

Figure 2 The geographic location of the side slope in the research area

2. Basic theory behind the linear regression method

2.1 Establishing a linear regression equation with one variable

Unitary linear regression can be used to estimate two further variables from one of two related variables. The essence of regression analysis is to find a mathematical model for fitting existing data that can be expressed in the form $y = f(x)$. For the independent variable, x_i, the values of the dependent variable, y_i, are not uniquely determined. Instead, they are distributed above and below a straight line and can usually be represented by a random perturbation term. Thus, the relationship between the independent variable and the dependent variable can be written

$$y_i = \alpha + \beta x_i + \varepsilon_i. \quad (1)$$

To perform a regression analysis, the zero mean value, homovariance, and independent of each other are made to conform to a normal distribution. The mean value, $E(y_i)$, can be written as

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$$E(y_i) = \alpha + \beta x_i, \tag{2}$$

where α is a constant term and β is the regression coefficient for the population, The two parameters of and in the regression equation are impossible to observe directly and need to be analyzed in combination with the collected sample data. Samples from the data can be used to estimate the values of α and β. Two parameters, 'a' and 'b', can be used to derive the sample regression equation:

$$y_i{}^\wedge = a + bx_i. \tag{3}$$

Here, y_i is the estimated value of the dependent variable, $E(y_i)$, that corresponds to the independent variable, x_i, The prediction model for unitary linear regression can be obtained by using values of a and b that are based on actual, observed data.

2.2 Parameter estimation for a unary linear regression equation

The coefficients a and b are determined and then used to obtain the vertical distance between each point in the scatter diagram of the acquired data and the regression line that has been obtained. The regression equation should satisfy the condition that the squared sum of the deviations for all of the collected data is a minimum: i.e.,

$$\sum_{i=1}^{n}(\boldsymbol{y_i} - \boldsymbol{\hat{y}_i})^2 = \sum_{i=1}^{n}[\boldsymbol{y_i} - (\boldsymbol{a} + \boldsymbol{bx_i})]^2. \tag{4}$$

The least squares method is used to estimate the values of a and b in the regression equation, and the differential method can be used to obtain the following equation:

$$\begin{cases} \sum y_i = na + b\sum x_i \\ \sum x_i y_i = a\sum x_i + b\sum x_i^2 \end{cases}. \tag{5}$$

The corresponding estimates, usually expressed as a and b, can then be obtained from the above equations:

$$\begin{cases} b = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \\ a = \frac{\sum y_i}{n} - b \times \frac{\sum x_i}{n} = \bar{y} - b\bar{x} \end{cases}. \tag{6}$$

2.3 Standard error of the linear regression estimation

When setting up a linear regression prediction model, generally the standard error is used to verify the accuracy of the regression equation; the estimated standard error describes the amount of deviation between the real and estimated values of the variables. The smaller the value of the estimated standard error, the closer the estimated data are to the real data. The actual and estimated values can then be used to set up a linear regression equation with k independent variables, where the number of degrees of freedom, $df = n - k - 1$:

$$\boldsymbol{S_y} = \sqrt{\frac{\boldsymbol{SSE}}{\boldsymbol{n-k-1}}}. \tag{7}$$

SSE is the sum of the squares of the deviations between the actual values and the estimated values.

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3. Establishment of an empirical model

3.1 Data collection and analysis

The study area was located in a zone with a changeable climate, and so it was particularly important to collect monitoring data during the rainy season. Multiple rainfall and displacement data were collected each month during the rainy season to provide reliable data for subsequent analysis. In this study, data from July were selected for further analysis.

At the monitoring point on the Tianmaogou landslide, the rainfall is concentrated in July. At this time, the stability of the landslide is greatly reduced, and the correlation between the amount of rainfall and the displacement is high. Accordingly, the results of the preliminary analysis showed that the three-dimensional displacement and rainfall have a linear relationship at this stage.

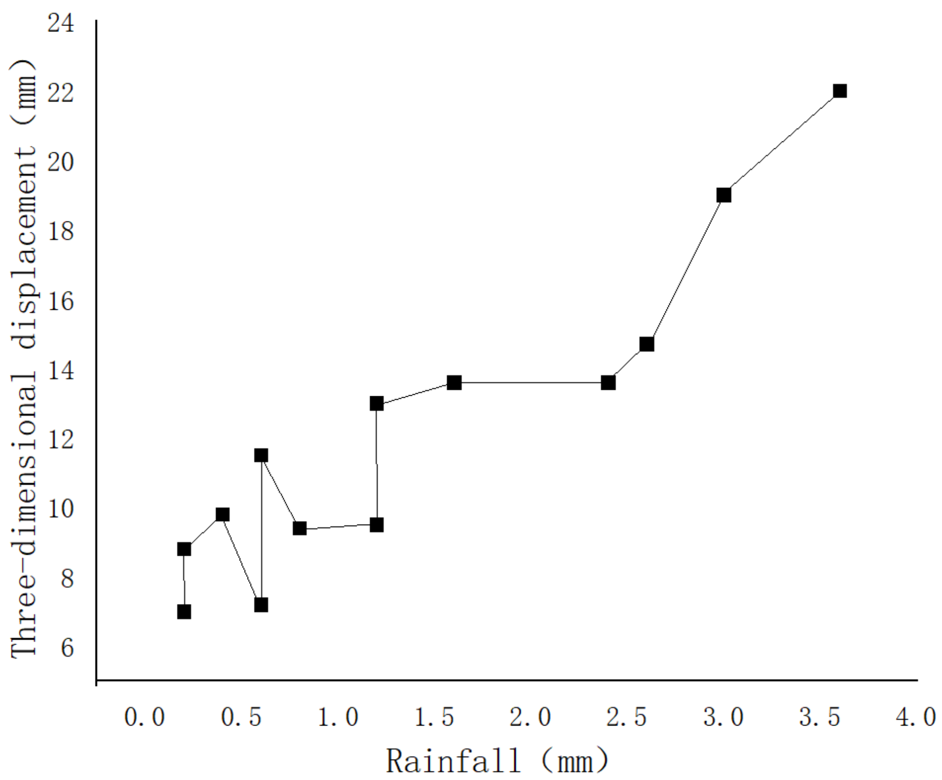

Figure 3 Rainfall and 3D displacement data graph

3.2 Linear regression modeling

Linear regression analysis was then performed using the acquired data, and the results are shown in Figure 3. By estimating the degree of correlation between the variables and the regression equation and by calculating the estimated standard error, a displacement–rainfall linear regression model for July was obtained. Further details of this are shown in Table 1. It can be seen that the two variables are closely correlated and that the regression equation is highly accurate.

Table 1 Linear regression calculation result

SSE	Sy	empirical model
26.141	1.542	Y=3.648x+7.07608

In summary, the empirical linear regression model for Tianmopou in July fits the observed data well.

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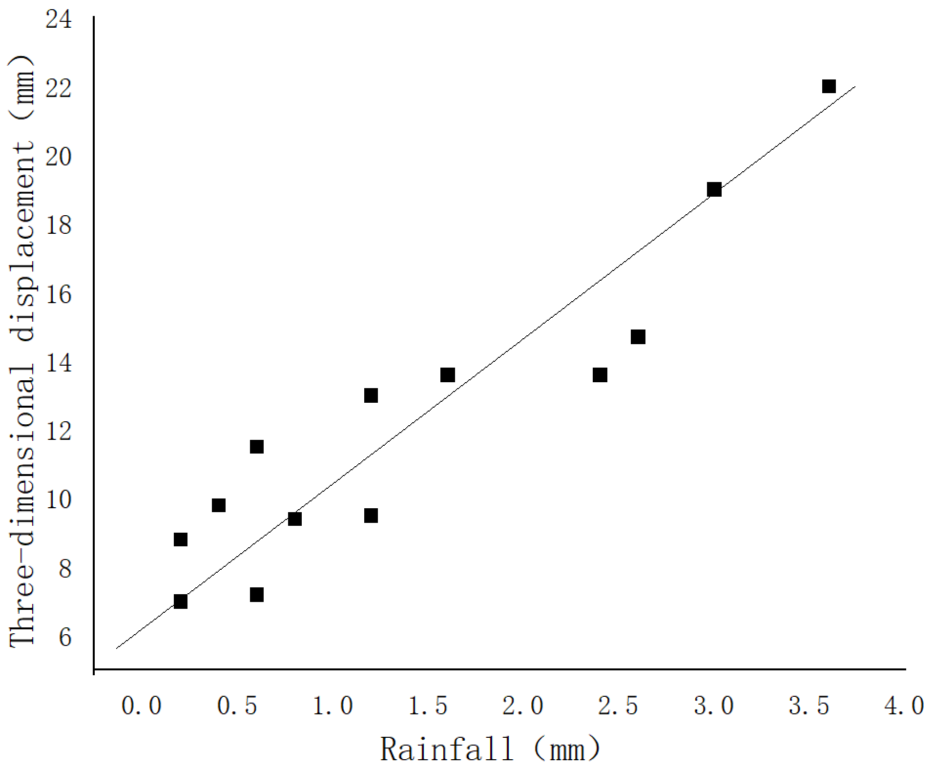

Figure 4 Straight line graph of rainfall and 3D displacement

4. Conclusion

(1) Due to the effects of the rainfall that occurs in July, the three-dimensional displacement of the slope at Tianmaogou, Bomi County, Tibet Autonomous Region, China tends to increase as the amount of rainfall increases. If there is sufficient continuous rainfall, there is a risk of landslides occurring.

(2) Data collected during July at Tianmaogou were selected for analysis. The results showed that the rainfall and the three-dimensional displacement were positively correlated and that the linear regression model that was developed agreed with the trends in the data and could be used for reference.

(3) In this study, the precipitation and three-dimensional displacement were used as variables in the regression analysis. The difference between the calculated values and the original monitoring data in the later stage was less than 2 mm, and the accuracy was relatively high. The model that was developed could thus provide a theoretical basis for landslide monitoring and prevention at Tianmaogou.

References

[1] Peng Cui, Xiaoqing Chen, Zunlan Cheng, et al. Monitoring and Prevention of Debris-flows and Landslides in Tibet[J]. Chinese Journal of Nature, 2010,32(01):19-25+66.

[2] Yongdong Jiang, Xuefu Xian, Jiang Xu, et al. Discussion on the Influence of Rainfall and Groundwater on Slope Stability in the Three Gorges Reservoir Area[C]// Chinese Society of Mechanics, Beijing University of Technology. Academic Conference of Chinese Society of Mechanics '2005 Abstracts of Papers (Part 2). 2005:708.

[3] Shanxiong Chen, Xin Tan, Zhiguo Liu.A prediction method of slope stability due to permeation of rainwater[J]. Rock and Soil Mechanics, 2002,23(S1):31-36.

[4] Yonglei Zhang, Xiufeng He, Zhiping Liu. Study of slope displacement prediction model based on empirical mode decomposition[J]. Yangtze River, 2013,44(03):48-51+92.