

# Summary of Research on Multi-Objective Optimization Problems

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## Abstract

Multi-objective optimization problems exist everywhere in life, and solving these problems can bring many benefits to society. Multi-objective optimization is to optimize all the objective functions in the problem, but in general, there is a little conflict between these objective functions, so it is difficult to achieve multi-objective optimization. So far, many experts and scholars have carried out research on multi-objective optimization problems and have achieved a series of results. This article starts from the basic description of multi-objective optimization problems and the research status so far. There are many methods to solve multi-objective optimization problems, such as objective optimization method. Such traditional methods as well as intelligent algorithms such as particle swarm optimization and differential evolution algorithm provide a comprehensive overview of multi-objective optimization problems.

## Keywords

Multi-objective optimization, intelligent algorithm, traditional optimization method.

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## 1. Introduction

In theoretical research and engineering applications, many specific problems can be attributed to an optimization problem. The research on this kind of optimization problem has always attracted the attention of many researchers. In real life, most of the problems discuss the optimization of multiple goals. Since the 1960s, many experts and scholars have begun to pay attention to this kind of problem, because it exists widely in our lives, and solving this kind of problem can solve many problems in life and industry [1].

The main difficulty of the multi-objective optimization problem is to achieve the optimal value of multiple objectives at the same time, so the multi-objective problem is much more complicated than the single-objective problem. However, the objective function between multiple objectives is often not highly adaptable, so it is difficult to solve the multi-objective problem. After the efforts of some experts and scholars, many multi-objective optimization methods have emerged, such as traditional multi-objective optimization methods such as  $\epsilon$ -constraint method or intelligent multi-objective algorithms such as differential evolution algorithm [2].

## 2. The relevant theoretical background of multi-objective optimization

### 2.1 Basic description of multi-objective optimization problem

The difference between a multi-objective problem and a single-objective problem is that it often has multiple objective functions, and optimizing it is to make its solution satisfy all objective function optimization. In multi-objective optimization problems, often only one solution that satisfies one of the objective functions can be found, and it is basically impossible to make it satisfy all objective functions. Therefore, a moderate feasible solution set is produced to make all the objective functions have better values. We call this solution set the Pareto optimal solution set [3,4]. The Pareto optimal

solution set forms the optimal boundary of the multi-objective optimization problem, so solving the multi-objective optimization problem is to find its optimal boundary.

Define a multi-objective optimization problem as follows [5]:

$$\begin{aligned} \min f(x) &= [f_1(x), f_2(x), \dots, f_n(x)]^T \\ \text{s.t. } g_i(x) &\leq 0 \quad i = 1, 2, \dots, j \\ h_i(x) &= 0 \quad i = j+1, \dots, p \end{aligned} \quad (2-1)$$

In the formula,  $x = (x_1, x_2, \dots, x_n)^T$  is an n-dimensional vector in  $R^n$  space, Call the space where  $x$  is the decision space of the problem,  $f_i(x) (i = 1, 2, \dots, n)$  is sub-objective function, n-dimensional vector  $f(x) = [f_1(x), f_2(x), \dots, f_n(x)]^T$  is called the target space of the problem,  $g_i(x) \leq 0 (i = 1, 2, \dots, j)$ ,  $h_i(x) = 0 (i = j+1, \dots, p)$ , they are inequality and equality constraint functions.

What multi-objective optimization needs to solve is to obtain good solutions for all the objective functions in the problem. The optimal solution is the Pareto optimal solution filtered through the Pareto dominance relationship. Often there is more than one Pareto optimal solution, and the optimal solution set of a multi-objective optimization problem is a set containing all Pareto optimal solutions.

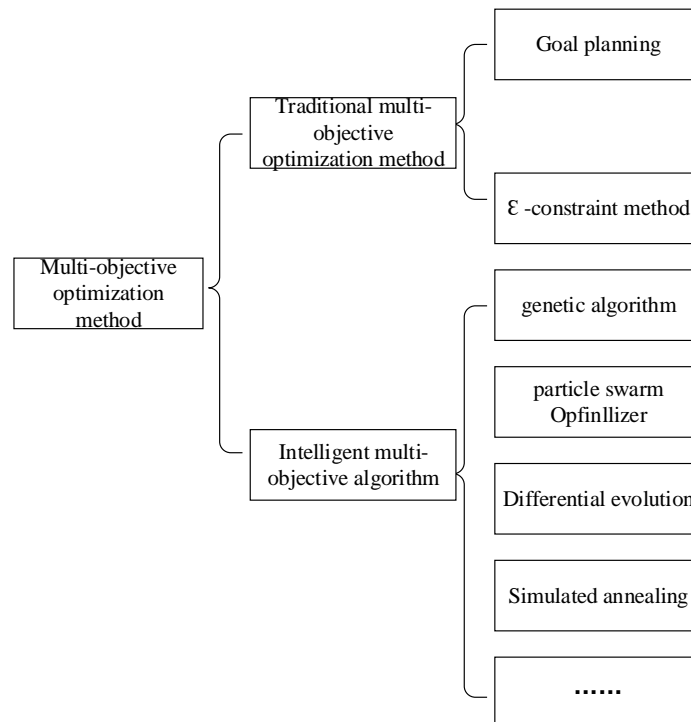


Figure 1. Classification of multi-objective optimization methods

## 2.2 Research status of multi-objective optimization problems

Multi-objective optimization problems are pervasive in our lives and industries. How to solve these problems is also the concern of many experts and scholars, so people put a lot of energy to find solutions. The method of multi-objective optimal decision-making was first proposed by Pareto from a mathematical point of view, and introduced the concept of Pareto optimal solution. This method can solve the multi-objective optimization problem very well. So far, people gradually began to pay attention to the field of multi-objective optimization [6]. Since the 1990s, many experts and scholars have put forward their own solutions to the multi-objective optimization problem, including the goal planning method,  $\epsilon$ -constraint method and some traditional optimization methods. Since the 1980s, with the rise and development of many intelligent algorithms, many researchers have begun to use evolutionary algorithms and particle swarm algorithms to solve such problems. Because of the

parallel mechanism of this kind of method, it can optimize multiple targets at the same time, so many excellent methods have appeared in this period [7,8]. Figure 1 shows a lot of methods for multi-objective optimization, as shown in the figure below:

### 3. Introduction to multi-objective optimization methods

#### 3.1 Multi-objective optimization traditional method

The objective programming method begins to add the expected value given by each objective function to the optimized objective function. Expressed in symbolic language as: for each optimization objective function  $f_i(x)$  gives a target expected value  $d = [d_1, \dots, d_m]^T$ ,  $i = (1, \dots, m)$ , then the multi-objective optimization problem can be transformed into:

$$\min f(x) \rightarrow \min \sum_{i=1}^m |f_i(x) - d_i| \quad (3-1)$$

Among them,  $d_i$  corresponds to the expected value of the  $i$ -th target. The applicable condition of this method is that the expected value is in the feasible region, so that the Pareto optimal solution can be obtained. Jie Yuan et al [9] used the goal planning method to optimize the unit line of the time period. It was found that this method successfully suppressed the zigzag phenomenon of the unit line of the time period, and the calculated ground runoff was in good agreement with the observed value, and compared with the least two Multiplication, using the goal planning method to calculate the unit line of the time period is smoother, there is no sawtooth phenomenon, can more truly reflect the hydrological response of the watershed to the rainfall input. The difficulty of this method lies in how to obtain the expected value of the objective function, which can solve linear problems, but for nonlinear problems, the results obtained are somewhat unsatisfactory.

The principle of  $\varepsilon$ -constraint method is to take out the most important objective function for optimization, and then treat other objectives as constraints:

$$\begin{aligned} \min f(x) &\rightarrow \min f_k(x) \\ \text{s.t. } f_i(x) &\leq \varepsilon_i, 1 \leq i \leq m, i \neq k \end{aligned} \quad (3-2)$$

Among them, the upper limit of the estimated value of the objective function must be given in advance. This method is to obtain a different solution according to each value. Jie Liang et al [10] proposed to use the  $\varepsilon$ -constraint method and the dictionary optimization method to gradually identify the redundant area of the dictionary based on the  $\varepsilon$ -expected value formed in the initial process of the  $\varepsilon$ -constraint method iterative optimization, and to update the corresponding elements of the income matrix in the narrowed search range further search within, improve the search efficiency. This optimization method is relatively simple and easy to implement, but the selection of parameter values will affect the quality of the solution to a large extent. If it is too small, it may not find a feasible solution, and if the value is too large, it will lead to other objective functions. The value loss is too large, so it is difficult to determine the upper limit of the estimated value of the objective function.

#### 3.2 Multi-objective intelligent algorithm

Two traditional multi-objective optimization methods are introduced above. It can be seen that these traditional methods are relatively simple and efficient, and have also been applied in the life industry. However, there are still many problems when solving multi-objective problems: 1) Traditional methods can only get one Pareto solution at a time when solving multi-objective problems, and the setting of parameter values will affect the quality of the obtained solution. Therefore, it is necessary to run the program multiple times and obtain more solutions by continuously adjusting the parameters. Each time the program is run, different results will be obtained, making it difficult for the decision maker to decide which solution is better, which greatly reduces the optimization efficiency. 2) These methods are not suitable for solving the problem that the optimal front segment is not convex. In contrast, some intelligent multi-objective algorithms can solve these problems well.

### 3.2.1 Genetic algorithm

When genetic algorithm (GA) was proposed by Dr. J.H. Holland in the United States, the reason for its slow development was that it did not attract the attention of academia. After the rise of artificial intelligence and the maturity of computer technology, genetic algorithms have slowly begun to be applied in the field of artificial intelligence, and they are also widely used in industry [11]. The process of genetic algorithm is actually a process similar to biological evolution in biology. At each iteration of genetic algorithm, excellent individuals are selected according to the fitness function, and new individuals are obtained by cross-mutating them. The optimal solution ends. Finally, the obtained population is decoded and the approximate optimal solution is obtained. Mirza Muntasir Nishat et al [12] use genetic algorithm to optimize the PID controller's DC-DC Buck converter. They use the optimized PID controller to control the traditional buck converter, and through the analysis of its performance parameters, the genetic algorithm (GA) is used to determine the key parameters of the converter. Tuning is carried out to improve its performance, and its stability is analyzed and researched. It is found that PID controller based on genetic algorithm is more convenient than other Buck converter tuning methods.

### 3.2.2 Particle Swarm Optimizer

The particle swarm Optimizer (PSO) is a newly emerged swarm intelligence optimization algorithm. It is very simple to adjust parameters and very convenient for computer processing. However, when the particle speed in the algorithm is too large, it will exceed the maximum. The solution of the optimal area, the result becomes divergent and the convergence is not optimal. If you adjust the inertia weight from the adaptive aspect, you can solve this problem well [13]. Yi Liu [14] Proposed a Pareto-based multi-objective particle swarm algorithm. The traditional particle swarm algorithm has the shortcoming of being easy to be premature. So she joined the variable neighborhood search strategy and proposed a variable neighborhood multi-object particle swarm algorithm to solve multi-objectives, supply chain distribution network optimization problem. Qinyue Zhu et al [15] in order to solve the problem that the midpoint potential imbalance of the midpoint clamped three-level traction inverter and the resulting output current harmonics cannot be effectively controlled at the same time, a traction inverter based on the particle swarm algorithm is proposed. Multi-objective optimization control strategy of the device. After optimization by the particle swarm algorithm, the amplitude of the midpoint potential fluctuation is minimized while effectively suppressing the output current harmonics.

### 3.2.3 Differential evolution algorithm

Differential evolution (DE) algorithm is a heuristic algorithm based on population evolution proposed by Storn.R and Price.K in 1995. It is a random parallel global search algorithm. It can remember the optimal solution of the individual and can share information in the group. Compared with other algorithms, the differential evolution algorithm has higher stability and faster convergence speed. Differential evolution algorithm has mutation, crossover and selection operations. Its unique differential mutation operation can ensure the diversity of the population and enable the population to develop in a better direction [16]. Suryatmojo Heri et al [17] Optimize the design of DFIG wind turbines. Because DFIG wind turbines have variable frequency AC/DC/AC converters (VFC) and IGBT switches, they are very sensitive to overcurrents that occur during transient disturbances. In order to protect the DFIG rotor circuit from excessive current, the optimal design of the PI controller in the DFIG wind turbine rotor-side converter (RSC) using the differential evolution algorithm (DE) is proposed to improve the DFIG's performance during disturbances. performance. After the simulation experiment, compared with the initial controller design, the overshoot rotor current and voltage DC-link of DFIG are significantly reduced.

### 3.2.4 simulated annealing

Simulated annealing (SA) is an optimization algorithm proposed based on the annealing process of metal solids in physics. The annealing of metal solids is divided into three processes: heating process, isothermal process and cooling process [18]. Although the calculation process of the SA

algorithm is simple and robust, it is easily affected by the initial parameters and cooling strategy, and the convergence speed is very slow. Jinwen Li et al [19] in order to solve the problem of route customization for new tourists when traveling in unfamiliar cities, a tourism route customization model with tourism utility value as the objective function is established under the constraints of distance from scenic spots, tourism consumption and tourist travel time. This improved algorithm uses chaos to find The initial temperature is optimally determined to avoid iterative redundancy; the search step is dynamically controlled by chaotic disturbance to the current optimal solution to jump out of the local optimal; and the variance judgment criterion is used as the search stop condition. Xing Wan et al [20] in order to improve the problems of slow convergence and too long time-consuming of simulated annealing algorithm, based on the basic principles of simulated annealing algorithm, aiming at the convergence problem of simulated annealing algorithm, a simulated annealing algorithm with memory is proposed to solve the TSP problem. After the algorithm was improved, considering the problem of long search time, they proposed a parallel design of MPI based on the improved algorithm to improve the search efficiency of the improved simulated annealing algorithm and reduce time consumption.

#### 4. Conclusion

There are many multi-objective optimization problems in life and industry. The multi-objective optimization problem has become a difficult problem that many experts and scholars are struggling to overcome. This article starts from the basic description and research status of the multi-objective optimization problem, and then solves the multi-objective so far. Traditional methods of optimization problems like  $\epsilon$ -constraint method and intelligent algorithms like differential evolution algorithm are explained. With the efforts of experts and scholars, it is believed that in the near future, there will be more powerful algorithms with superior performance.

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