

A Structural Topology Optimization Method based on SIMP and Conductive Weight Method

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Abstract

With the research and development of topology optimization methods, in order to improve the engineering manufacturability of optimized structures, some scholars have proposed different methods. In this paper, the optimization criterion method is replaced on the basis of SIMP, and the guide weight method is used, and the optimization criterion method is compared. Firstly, the Lagrange function is performed on the continuous topology optimization model to solve the problem of least flexibility. Then, the density update is performed on the Guide-weight method and the optimization criterion method. Finally, the Hesviside function is used for filtering to avoid the generation of grayscale elements. After a two-dimensional study and a three-dimensional study, it is concluded that the engineering manufacturability of the optimized structure obtained by the guide method is better than that obtained by the optimization criterion method.

Keywords

SIMP; Guide-weight Method; Topology Optimization.

1. Introduction

Topology optimization is based on the given load conditions, constraints and performance indicators, the material distribution of the structure is optimized in a certain design domain to achieve the effect of lightweighting the structure while ensuring the performance of the structure.

As one of the most cutting-edge methods of modern structural design, structural topology optimization plays an important role in engineering applications. Many scholars have studied different topology optimization methods in order to achieve a stable and efficient topology optimization method with practical engineering value. Erik [1] proposes a more efficient topology optimization method on the basis of variable density method; Ferrari [5] proposed a faster calculation method based on 99 lines of code, and studied 3D topology optimization; [2-4] proposed a variety of efficient structural topology optimization methods. In this paper, the original optimization criterion method is replaced on the basis of the SIMP method, and the Guide-weight method is used to solve the SIMP model, which can not only solve the topology optimization problem more efficiently, but also produce an optimization structure with more practical design significance.

2. The Guide-weightmethod Solves the Topology Optimization Problem of the SIMP Model

The specific idea of the SIMP model used in this paper is to change the Young's modulus of the unit through the interpolation function to generate intermediate elements. The SIMP method is an

interpolation function constructed between the relative element density and the young elastic modulus of the element. The interpolation function expression is given as follows:

$$E_e = E_e(x_e) = x_e^p E_0 \quad x_e \in [0,1] \quad (1)$$

where p is the penalty factor.

In order to avoid the singularity of the stiffness matrix during the calculation process, an improved SIMP density function is used as:

$$E_e = E_e(x_e) = E_{\min} + x_e^p (E_0 - E_{\min}) \quad x_e \in [0,1] \quad (2)$$

The purpose of using the SIMP model is to map the $\{0,1\}$ discrete variables into continuously differentiable $[0,1]$ continuous variables. The density value of $(0,1)$ is punished by using the penalty function, and the continuous variable values are approximated to the discrete variable of $\{0,1\}$ to eliminate the intermediate variable.

In the topology optimization problem of the continuum, the solution of the simplex problem can generally be divided into two methods to build the optimization model: (1) Find the minimum flexibility under the mass constraint; (2) Find the minimum mass under the displacement constraint. In this paper, the topology optimization problem of the continuous is solved to find the minimum flexibility under the quality constraint, and then the SIMP method is used to establish the material interpolation model of topological optimization, and the optimization model of the topology optimization of the continuous problem is:

$$\begin{aligned} &\text{Find } X = (x_1, x_2, \dots, x_i, \dots, x_N)^T \in \mathbf{D} \\ &\min C(X_e) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N E_e(X_e) \mathbf{U}_e^T \mathbf{K}_0 \mathbf{U}_e \\ &\quad s.t. V(X_e) / V_0 = f \\ &\quad \quad \mathbf{K} \mathbf{U} = \mathbf{F} \\ &\quad \quad 0 \leq X_e \leq 1 \end{aligned} \quad (3)$$

Based on the mathematical model of the topology optimization of the above continuum, the Lagrange function can be constructed as follows:

$$\begin{aligned} L(x, \lambda) = & C(x) + \lambda_e (V - fV_0) + \lambda_d (d_j - d_0) + \lambda_w (\omega_0 - \omega_n) + \\ & \lambda_1^T (\mathbf{K} \mathbf{U} - \mathbf{F}) + \sum_{e=1}^N \lambda_2^e (x_{\min} - x_e) + \sum_{e=1}^N \lambda_3^e (x_e - 1) \end{aligned} \quad (4)$$

where λ is a Lagrange multiplier.

According to the Lagrange equation above, the Lagrange function satisfies the Kuhn-Tucke condition when solving the optimal solution to the minimum flexibility optimization problem:

$$\begin{aligned} \frac{\partial L}{\partial x_e} &= \frac{\partial C}{\partial x_e} + \lambda_v \frac{\partial V}{\partial x_e} + \lambda_d \frac{\partial d_j}{\partial x_e} + \lambda_\omega \frac{\partial \omega_n}{\partial x_e} + \\ &\lambda_1^T \frac{\partial(\mathbf{KU})}{\partial x_e} - \lambda_2^e + \lambda_3^e = 0 \end{aligned} \quad (5)$$

In the solution process, the density update formula needs to change with the optimization model, so the Guide-weight method (GW method) is more versatile than the optimization criterion method, and both belong to the criterion method. According to the Kuhn-Tucke condition, the density update formula of the Optimization criterion (OC) method and the GW method can be respectively obtained as:

$$x_e = \begin{cases} \max(0, x_e - m) & , x_e B_e^\eta \leq \max(0, x_e - m) \\ \min(1, x_e + m) & , x_e B_e^\eta \geq \min(0, x_e + m) \\ x_e B_e^\eta & , \text{otherwise} \end{cases} \quad (6)$$

$$x_e = \begin{cases} 0 & , x_e \leq 0 \\ 1 & , x_e \geq 1 \\ m[G_e / (\lambda H_e)] + (1 + mx_e) & , \text{otherwise} \end{cases} \quad (7)$$

where m is the moving step, η is the damping factor;

$B_e = \frac{-\partial c / \partial x_e}{\lambda \partial V / \partial x_e}$ is the heuristic update factor;

$G_e = -x_e \frac{\partial c}{\partial x_e}$ is the element guide;

$H_e = \frac{\partial V}{\partial x_e}$ is the element capacitance;

Since the SIMP variable density method is used to solve the optimization problem of the extended continuum, a large number of gray element will be generated in the final optimized structure, which will cause the boundary of the structure to be blurred. Therefore, some measures are taken to prevent the generation of grayscale elements and improve the clarity of the boundaries of the resulting topology optimization structure. The Heaviside projection function can have a good inhibition effect on gray element, and can reduce the number of gray element in the structure to obtain a topological optimization structure with clear boundaries. Therefore, a Riverside projection function is used to filter the guide method and the optimization criterion method. The Heaviside projection function is given as follows:

$$\begin{cases} \text{Function: } x_e^{new} = 1 - \exp(-\beta \hat{x}_e) + \hat{x}_e \exp(-\beta) \\ \text{Derivative: } \frac{\partial x_e^{new}}{\partial \hat{x}_e} = \beta \exp(-\beta \hat{x}_e) + \exp(-\beta) \end{cases} \quad (8)$$

When filtering gray element by the Heaviside projection function, the sensitivity of volume and flexibility is given as:

$$\frac{\partial \phi}{\partial x_j} = \sum_{e \in N_j} \frac{\partial \phi}{\partial \hat{x}_e} \frac{\partial \hat{x}_e}{\partial x_e} = \sum_{e \in N_j} \frac{1}{\sum_{i \in N_e} H_{ei}} H_{je} \frac{\partial \phi}{\partial x_e^{new}} \frac{\partial x_e^{new}}{\partial \hat{x}_e} \quad (9)$$

where x_e^{new} is the density of the elements after filtering the grayscale elements by the Heaviside projection function.

3. Study Validation

3.1 Two-dimensional Study

The simplified schematic diagram of the two-dimensional topology optimization structure of the simple support beam is shown in Fig. 1, and the optimized discrete element of the simple support beam is set to 120 in length and 60 in width. Set the concentrated load in the middle of the lowest end of the simple support beam.

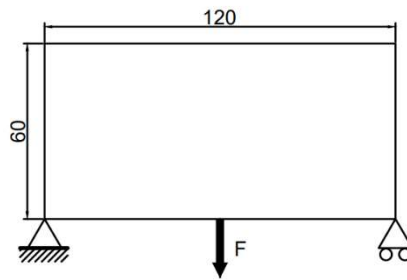


Fig. 1 A two-dimensional diagram of a simple support beam

Set the parameters of the upper limit of Young's modulus $E_0=1$, the lower limit of $E_{min}=1e-9$, the concentration size $F=1$, the volume ratio $f=0.5$, the filter radius $r_{min}=2.5$, and the penalty factor of SIMP is given as: $p=3, \beta=1$.

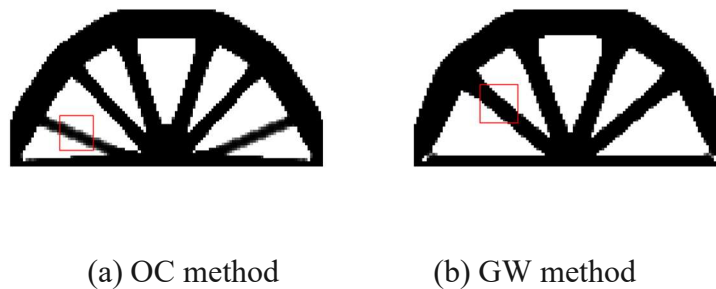


Fig. 2 Structural optimization diagram of two-dimensional simple support beam

As shown in Fig. 2, both the OC method and the GW method can produce usable optimized structures, compared to (a) and (b) in Fig. 2, the optimized structure solved by the OC method will produce members with gray element boundaries, while compared to the OC method, the GW method produces structures, there is no member with gray element, which is more concise and easy to process.

Table 1. Comparison of data after optimization of two-dimensional simple beams by OC method and GW method

	Method	Steps	Flexibility
Equation 9	OCmethod	9392	14.4448
Equation 9	GWmethod	10470	14.3624

From Table 1, it can be seen that when solving the problem of topology optimization of the simple continuum with the goal of minimizing flexibility, comparing the OC method and the GW method, it can be seen that the number of iteration steps required by the OC method is smaller than that of the GW method, but the optimization speed is limited; the flexibility obtained by the GW method is slightly less than that of the OC method; combined with the number of iteration steps, the flexibility and the analysis is shown in Fig. 2, the OC method is similar to the solution speed and accuracy of the GW method, but the GW method can get a better structure than the processing, which is more helpful for the actual engineering design.

3.2 3D Study

A brief diagram of the three-dimensional topology optimization structure of the cantilever beam is shown in Fig. 3, and the discrete unit of the cantilever beam optimization is 40 in length, 5 in width, and 10 in height. Set the concentrated load in the lowest middle position of the unsettled end of the cantilever beam.

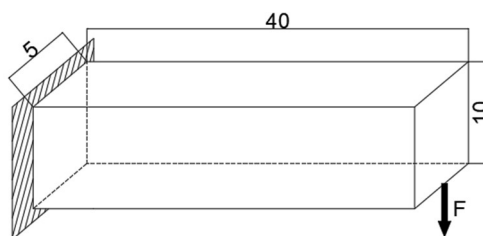
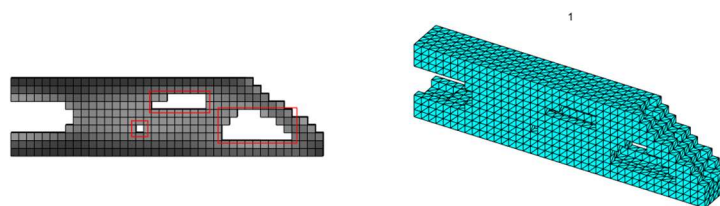
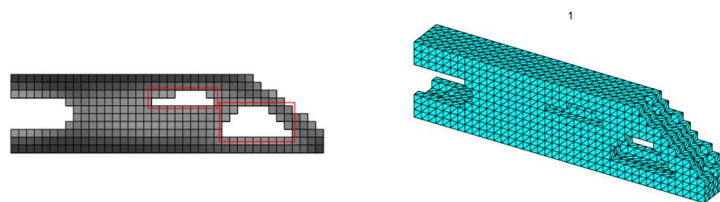


Fig. 3 3D sketch of a cantilever beam

Set the parameters of the upper limit of Young's modulus $E_0=1$, the lower limit of $E_{min}=1e-9$, the concentration size $F=1$, the volume ratio $f=0.5$, the filter radius $r_{min}=2.5$, the penalty factor of SIMP $p=3, \beta=1$.



(a) OC method



(b) GW method

Fig.4 Structural optimization diagram of a 3D cantilever beam

As shown in Fig. 4, both the OC method and the GW method can produce usable three-dimensional optimized structures, compared with (a) and (b) in Fig. 4, the three-dimensional optimized structure solved by the OC method will produce fine holes that are not easy to process, while compared to the OC method, the three-dimensional structure produced by the GW method does not have a fine pore structure, and the integrity of the structure is better and easier to process.

Table 2. Comparison of data after optimization of 3D cantilever beams by OC method and GW method

	Method	Steps	Flexibility
Equation 9	OCmethod	3520	4240.1612
Equation 9	GWmethod	3516	4250.2475

From Table 2, it can be seen that when solving the problem of topology optimization of the simplec case continuum with the goal of minimizing flexibility, comparing the OC method and the GW method in the three-dimensional structure, it can be seen that the number of iterative steps required by the OC method is greater than that of the GW method; the flexibility obtained by the GW method is slightly greater than that of the OC method, combined with the flexibility and iterative steps, the OC method and the GW method have their own advantages and disadvantages in terms of the accuracy of the solution speed; combined with the analysis of Fig. 4, the three-dimensional optimization structure obtained by the GW method is better than that obtained by the OC method. Therefore, the GW method is more helpful for practical engineering design.

4. Conclusion

By comparing the OC method and the GW method with a three-dimensional simple support beam study, the following conclusions can be drawn from the topological optimization problem of solving the SIMMP model by comparing the OC method and the GW method: (1) the OC method and the GW method have the following conclusions in solving the simplexage problem, whether it is a two-dimensional structure or a three-dimensional structure, the solution speed and the accuracy of the solution are similar; (2) the GW method is better than the OC method, whether it is a two-dimensional structure or a three-dimensional structure, and the final optimized structure is better; (3) For practical engineering design problems, the optimized structure of the GW method is stronger than the OC method.

Acknowledgments

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