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The Application of Function Thought in High School Mathematics Problem Solving

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Abstract

Consciously infiltration of function thought in teaching can spread students 'thinking, effectively help students to reduce the difficulty of the problem when solving problems, improve the efficiency of problem solving and cultivate students' good thinking quality and modeling ability. This paper starts from the concept of function, introduces the idea of function thought and shows the specific application of function thought in equation, inequality, sequence and geometry, and discusses the wide application of function thought in high school mathematics problem solving to shows the importance of infiltrating function thought in elementary mathematics teaching.

Keywords

Function Thought; High School Mathematics; Problem Solving Method.

1. Introduction

After entering high school, in addition to mastering the knowledge of textbooks, students should have a deeper understanding of mathematical thinking and methods, and the nature and connotation of mathematics. In recent years, the selection of mathematics propositions for the college entrance examination has paid more attention to the mathematical thinking of high school students. The comprehensive application ability of the method. Functions run through all knowledge points in junior high school and high school, and are involved in the learning of various mathematics sections. In the stage of high school mathematics learning, we derived the concept of functions from sets, and later learned basic functions, exponential functions, Logarithmic functions, trigonometric functions, etc. If students can't see the relationship between these many knowledge points, it is difficult to understand and master so many functional knowledge in a short time, but if students can string these functional knowledge together , and connect with other knowledge, such as quadratic function and equation, inequality, exponential function and quadratic radical, square root, etc., you will find that these knowledge can be integrated into a complete knowledge system through functional thinking. Understand functions It can not only greatly reduce the complexity of the problem and optimize the problem-solving process, but also help students gradually diverge their thinking in the problem-solving process, and cultivate students' mathematical thinking ability.

2. Function Thinking Method

Function thought is a kind of mathematical thinking method, which is produced with the development of function concept. The correspondence between variables is the essential feature of function, so what function relationship describes is the dependence relationship between quantities in nature. The so-called Solving problems by using the functional thinking method is to abstract the essential characteristics of the mathematical problem to be sought, then establish the corresponding functional relationship, and finally use the basic properties of the function such as image, monotonicity and periodicity to solve the problem. The core of functional thinking is to construct appropriate auxiliary functions skillfully, so when solving math problems, if you want to apply functional thinking to solve problems more quickly, the key is to be good at discovering the conditions implied in the problems and then constructing functional formulas. And apply the relevant properties of the function to solve the problem.

2.1 Several Common Methods of Constructor

In the process of solving mathematical problems, in addition to some fixed formulas, we also need divergent creative thinking, both of which are indispensable. The key to applying functional thinking to solve mathematical problems is to construct a suitable auxiliary function. Therefore, the related properties of functions can be used to solve problems, such as the graph of functions, monotonicity, periodicity, and the relationship between roots and coefficients. However, there is no fixed mode for constructing auxiliary functions. In solving problems, how to construct functions, construct What kind of function is very important for the subsequent problem-solving process, which requires students to think and analyze more in the usual problem-solving training, improve their thinking conversion ability, and thus improve the problem-solving efficiency. The following is a brief introduction to several common method of the constructor.

(1) Direct construction method

Direct construction method is the relevant formula and the topic of the condition or conclusion direct construction function relation, and through the specific relation to solve the problem, for some is not particularly complicated regular inequality or array expression, can be directly set as a function expression, and then can use the function properties to solve.

For example, if the differentiable function was know $f(x) < f'(x)$ and $f(0) = 2$. The solution set of the inequality is $\frac{f(x)}{e^x} > 2$. The function can be directly constructed $g(x) = \frac{f(x)}{e^x}$, then $g'(x) = \frac{f'(x) - f(x)}{e^x}$, from $f(x) < f'(x)$ and $e^x > 0$, that is $g'(x) > 0$, $g(x)$ monotonically increasing on R, and $g(0) = \frac{f(0)}{e^0} = f(0) = 2$, so the solution set of the $g(x) > 2$ is $(0, +\infty)$.

(2) Difference construction

In solving derivatives, inequalities, comparing the size of rational numbers, and proving the monotonicity of functions, the difference construction method is the most commonly used and basic method. The general idea of this method is to construct functions $F(x) = f(x) - g(x)$, which can transform the problem into a Function $F(x)_{\min} \geq 0$ (or $F(x)_{\max} \leq 0$), that is, a common method for finding the maximum value of a function.

For example, for the problem of comparing the size of two rational numbers, you can first find the difference between the two numbers, and see whether the difference is greater than zero, equal to zero, or less than zero, so that the size of the two numbers can be determined. That is: if $a - b > 0$, then $a > b$; if $a - b = 0$, then $a = b$; if $a - b < 0$, then $a < b$.

For the problem of proving inequalities, it can also be proved by the difference method. For example, it can be proved that: when $x > 0$, for $\ln(1+x) > \frac{x}{x+1}$, it can be set $f(x) = \ln(1+x)$, $g(x) = \frac{x}{x+1}$, and then the difference can be made through the constructor function, that is $F(x) = f(x) - g(x) = \ln(1+x) - \frac{x}{x+1}$. Derivation of the above function. when $x > 0$, $F'(x) > 0$. It can be proved that the inequality holds $F(x) > F(0) = 0$.

(3) Local construction

If known a function $F(x)$ resolution looks more complex, whether guide or separation parameters will only make the topic become more complex and make problem is difficult to go on, then can function into $F(x) = f(x) \cdot g(x)$ (or $F(x) = f(x) + g(x)$), this will be much simpler than the original relationship, namely the original function do some local processing, to solve, will greatly reduce the problem difficulty, make the problem simple and easy to solve.

For example, it is known that $2a^2 - 5\ln a - b = 0$ the minimum value $\sqrt{(a-c)^2 + (b+c)^2}$ can be disassembled. The auxiliary function $2a^2 - 5\ln a - b$ can be constructed $f(x) = 2x^2 - 5\ln x, g(x) = -x$. The minimum value $\sqrt{(a-c)^2 + (b+c)^2}$ represents the distance from the tangent parallel $y = f(x) = 2x^2 - 5\ln x$ and $y = -x$. Finally use the distance formula between two parallel lines to find the answer.

(4) Swap construction

So-called change, popular is transformation, which is equal replacement, so when solving some mathematical exercises, for those more complex formula, you can be a part as a whole, and then use a variable to replace this part, after the deformation of the formula, can simplify the problem to solve, so that the problem can achieve simplification. However, it should also be noted to consider whether the value range of the variable changes after changing the element method. Through the element method constructor, we can simplify the operation, turn the high power into the low power, and turn the fraction into the integral formula, so for the more complex inequality, the expressions such as the series and the conical curve.

For example, for any positive integer n, when the inequality $\ln(\frac{1}{n} + 1) > \frac{1}{n^2} - \frac{1}{n^3}$ is verified, only by the order $\frac{1}{n} = x$, the inequality is converted to $\ln(x+1) > x^2 - x^3$, then the function can be constructed $h(x) = x^3 - x^2 + \ln(x+1)$, and then the derivative can achieve the proof.

Similarly, for formulas a_n containing columns $\frac{a_n}{2^n} = \frac{a_{n-1}}{2^{n-1}} + 1$, ordered $b_n = \frac{a_n}{2^n}$, then $b_{n-1} = \frac{a_{n-1}}{2^{n-1}}$, after expressions of columns b_n, namely $b_n = b_{n-1} + 1$. For elliptic expressions $\frac{r_1^2 \cos\alpha^2}{8} + \frac{r_2^2 \sin\alpha^2}{4} = 1$, make $x = r_1 \cos\alpha$, $y = r_2 \sin\alpha$, by changing elements $\frac{x^2}{8} + \frac{y^2}{4} = 1$.

(5) Logarithmic constructor

When a function is product, quotient form or power form, exponential form, can adopt logarithmic construction method, namely the two sides of the same time, the multiplication or division operation can be reduced to addition or subtraction operation, the power function, exponential function reduced into multiplication and division operation, to achieve the purpose of simplified operation.

For example, at that time $x > 0$, the inequality $(1+x)^{1+\frac{1}{x}} < e^{1+\frac{x}{2}}$ is proved, the inequality is in exponential form, so you can take the logarithm of the two sides of the inequality $(1+\frac{1}{x})\ln(1+x) < 1 + \frac{x}{2}$, simplified as $2(1+x)\ln(1+x) < 2x + x^2$, and then the inequality can be simplified through the constructor $f(x) = 2x + x^2 - 2(1+x)\ln(1+x), (x \geq 0)$, and then the inequality can be proved by useing the function derivative and the second derivative can be proved.

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3. The Application of Function Thought in High School Mathematics Problem Solving

3.1 The Application of Functional Ideas in Eq

In high school mathematics, we found through learning, although the concept of function and equation is different, but can not see them as independent knowledge, and to master the connection between them, such as for equation $f\left(x\right)=0$, its root can be regarded as a function $y=f\left(x\right)$ when 0 value, otherwise, function $f\left(x\right)-y=0$ zero, can also be regarded as the root of the equation, students if you can be familiar with the transformation between the two, can skillfully optimize the mathematical problem solving process.

Example 2.1 Take $x^2+ax+b^2-2=0$ $(a,b\in R)$ the equation on (-∞,-2)∪[2,∞) and find the value range a^2+b^2.

This question at first glance seems impossible to start, then can start from known conditions, according to the conditions of the equation of the two root in (-∞,-2)∪[2,∞), can get the conditions, will be seen as the area and the coordinates origin distance square $\begin{cases}2-2a+b^2\le 0\\ 2+2a+b^2\le 0\end{cases}$, and then through the number also can get the results, but the idea of problem solving process is more tedious. If the equation is regarded as $x^2+ax+b^2-2=0$ a line in the coordinate plane: $P(a,b)$ it is a point in the line $xa+b^2+x^2-2=0$, it is the value range a^2+b^2 based on the distance formula of the point line $\left|PO\right|^2$.According to the distance formula of the point line and the value range of the function monotonicity a^2+b^2 can be solved.

3.2 Application of Functional Ideas in Inequalities

Just as functions and equations can translate, functions and inequalities. For example, $f\left(x\right)>0$ for inequalities, they can be converted into the range of the current function $y>0$ $y=f\left(x\right)$. For some inequality problem, can also use the function thought, apply function thought to solve the inequality problem of one of the most important methods is to construct appropriate functions, but also should pay attention to in a inequality containing multiple variables, need to determine the appropriate variables and parameters to reveal the specific function relationship, make the problem clear and easy to understand.

Example 2.2 It is known that $f\left(t\right)=\log_2 t$, $t\in\left[\sqrt{2},8\right]$, for all real numbers in the value field, the inequality $x^2+mx+4>2m+4x$ holds constant, and that the value range of the x .

For this problem where the inequality is constant, it will be difficult to use the conventional problem solving idea. If the function idea is applied to construct the primary function and the original inequality can be transformed $\left(x-2\right)m+\left(x-2\right)^2>0$, the inequality can be regarded as the primary function processing $f\left(m\right)=\left(x-2\right)m+\left(x-2\right)^2$, so that the problem solving idea will be optimized, and the problem will be solved.

3.3 The Application of Function Thoughts in Arrays

In high school mathematics learning, array is a special function, its independent variable is a positive integer, the domain is a positive integer set and its subset, so the array problem can also be converted into functions to solve, columns and function can also achieve mutual conversion, such as series of general formula and previous term and formula can also be regarded as the expression of independent function relationship, the obvious function relationship provides another problem solving. In the process of answering some array questions, it is also a very effective way to use the function view to investigate, but also is conducive to students to better understand the meaning of sequence, general formula, but also can better grasp the difference, the monotony of the series and other related knowledge points, more effectively solve sequence exercises. The following analyzes the function ideas from two aspects of equal and equal columns.

(1) Function and arithmetic number columns

Example 2.3 The preceding $\{a_n\}$ sum of the known arithmetic number columns S_n, and $S_p = A$ $S_q = A$, ($p,q \in N^*$ 且$p \neq q$), ask S_{p+q}.

From the conventional point of view of problem solving, we can start from the arithmetic number column $S_p = pa_1 + \frac{p(p-1)}{2}d = qa_1 + \frac{q(q-1)}{2}d = S_q$, front item and formula, from the known and simplified $(p-1)a_1 + \frac{(p+q-1)(p-q)}{2}d = 0$, a_1 can be solved, and then into the sum formula S_{p+q} can get the results, but this solution method calculation is slightly complex, the method is more conventional and lack of skill, if through the construction of quadratic function, order $S_n = f\left(n\right) = an^2 + bn$, that is $f\left(p+q\right)$, you can use the quadratic function symmetry properties, this method is more simple, but also can exercise students' mathematical thinking ability.

3.4 Application of Functional Ideas in Geometry

In high school mathematics about geometry questions, the most value problem often appears in the conic comprehensive problem, has been a hot spot of the college entrance examination, and this type of topic is abstract and difficult to understand and complex calculation, students to solve this type of problem, in addition to master the conic related knowledge and problem solving skills, if can skillfully function ideas applied to them, will achieve twice the result with half the effort. The common idea of solving this type of problem is to construct a proper function relationship in the process of understanding the movement changes, and then solve the geometric most value by finding the function most value. In geometric problems, constructors can not only find the maximum value, but also is a common method to find area, range and other problems.

Example2.4: Known ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ （a＞b＞0）passes with eccentricity $(1, \frac{3}{2})$ of $\frac{1}{2}$.

(1)find the ellipse, the equation;

(2) Let the right vertex and right focus of the ellipse be points respectively, pass the straight line ellipse at two points, and find the maximum value of the quadrilateral area.

This is one of the most common questions in the college entrance examination mathematics, the first small question can be directly by pending coefficient method and eccentricity expression, and the second small ask quadrilateral area maximum, need to find out the expression of the quadrilateral area first, by drawing the sketch can be quadrilateral area into two triangle area, but the analysis topic can only get triangle bottom long and do not know how high, so change ideas, using the known conditions in the elliptic equation and straight line equation makes a new equation $(3k^2+4)y^2+6ky-9=0$,Therefore, the relationship between the ordinate can be found through Vader's theorem, and the quadrilateral area expression can be found $S_{OCAD} = S_{\vartriangle OCA} + S_{\vartriangle ODA} = \frac{1}{2}2\left|y_1\right| + \frac{1}{2}2\left|y_2\right| = \left|y_1 - y_2\right|$,After the simplification, the function can be passed monotone Find the area maximum value.

4. Attention to Paid When Applying Function Ideas

The application of function thought to high school mathematics problem solving can play a good effect, but in the practical application process, we should also pay attention to some matters.

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First of all, teachers should guide students to think independently in teaching, rather than only teaching conceptual theorems. Mathematical thought can not be understood by students through mechanical oral teaching, so it can be fully reflected in the exploration process of formulas and theorems, and gradually penetrate in teaching, so that students can understand the function thought more deeply.

Secondly, in addition to teachers need to pay attention to the infiltration process, students themselves should also strengthen their understanding of the nature of mathematical thought. Function thought is a kind of mathematical thought, and can not be directly obtained from the teaching material, but in learning and exercises, which is a subtle process. Therefore, in the usual mathematics learning, we should be good at summarizing, slowly understand the function thought in the learning process, and be good at applying it to problem solving and other learning, internalized into their own mathematical ability.

5. Summary

In short, function knowledge is the main line of high school mathematics knowledge, which plays an important role in the whole process of high school mathematics learning. The so-called function thought is a mathematical thought method to solve non-functional problems by constructing function analysis and then using function correlation properties. The application of function thought to mathematical problem solving can not only greatly reduce the complexity of the problem, optimize the problem solving process, but also help students to solve problems easily and effectively, gradually improve the comprehensive application ability of mathematical problem solving ability and thinking methods, and improve the effectiveness of learning.

For high school students, simply mastering the knowledge in the textbook can only cope with the exam, only by integrating knowledge points, understanding mathematical thinking, and encountering problems can really be learned.

References

[1] Fan Xuanfeng. The lever application of function thinking in high school mathematics problem solving [J]. Mathematical, physical and chemical problem solving research, 2020,10:11-12.

[2] Su Zhiqing. Use function thought to guide high school mathematics problem solving [J]. Mathematics, Physics and Chemistry Learning (Teaching and research edition), 2018,08:9-10.

[3] Haixiang Yu. Analysis of mathematical functions in middle school [J]. Science and Education Forum, 2017,11:29.

[4] Hu Qianfan. The Application of Test Theory Function Thought in High School Mathematics Problem Solving [J]. Teaching Frontier, 2018,02:46.

[5] Wang Changhu. The ingenious Application Analysis of Function Thought in High School Mathematics Solution [J]. National College Entrance examination, 2020,05:182.

[6] Pang Jinghong. On the Application of Mathematical Thought in High School Mathematics Problem Solving [J]. Modernization of Education, 2018,05 (27): 368-369.

[7] Xuemei Zhao. Study on the Strategies of Perenetrating Mathematical Thought in Analytic Geometry Teaching [J]. Mathematical, physical and chemical problem solving research, 2020,30:35-36.