

# Extreme Filtering Fuzzy C-Means Clustering Segmentation Method with Neighborhood Constraints

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## Abstract

Aiming at the problem that traditional fuzzy c-means (FCM) clustering algorithm is sensitive to noise in image segmentation, an extreme filtering fuzzy c-means with neighborhood constraints (EFFCM\_N) clustering segmentation method is proposed in this paper. The algorithm first filters the image, and then uses the processed image for fuzzy C-means clustering segmentation. In addition, the objective function is constrained by adding neighborhood information. Through the segmentation experiments of synthetic image and natural color image, the experimental results show that the proposed EFFCM\_N algorithm is obviously superior to the traditional FCM algorithm and some improved algorithms in segmentation effect and quality, and has stronger robustness to various noises.

## Keywords

Fuzzy C-means (FCM); Image Segmentation; Extreme Filter; Neighborhood Constraints.

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## 1. Introduction

Image segmentation has been widely used in various applications, such as robot vision, object recognition, geographic imaging and medical imaging [1]. Generally, image segmentation [2-6] is defined as dividing an image into non overlapping and consistent regions that are uniform with respect to certain characteristics (such as gray value or texture). Clustering usually distinguishes objects or patterns according to similarity measures (such as Euclid distance). In the clustering process, similar objects are usually grouped into the same cluster. Clustering is an important tool for pattern recognition and image analysis. It has been applied to medical image segmentation, remote sensing image classification, high-dimensional data clustering, infrared pedestrian segmentation and other fields. Among various clustering methods, FCM is a very classic and popular method in the industry due to its simplicity.

In 1973, Dunn proposed the FCM clustering algorithm based on the deterministic C-means clustering algorithm and fuzzy sets. Later, Bezdek promoted it [7]. FCM is one of the most commonly used image segmentation methods. Its success is mainly attributed to the introduction of fuzziness to achieve the attribution of each image pixel. Compared with clear or hard segmentation methods, FCM can retain more information in the original image. However, the standard FCM has a disadvantage that it does not consider any spatial information in the image context. In addition, there is always some distortion in the data acquisition and transmission process, which may lead to noise or outlier artifacts. Therefore, traditional FCM is very sensitive to noise. For the robustness of segmentation, many modified FCM algorithms are proposed. In order to solve the problem of noise and outliers, Zang et al. [8] proposed a kernel based intuitive fuzzy C-means clustering algorithm, namely KIFCM. This method mainly defines a local strength variance to suppress noise. Later, Bai et al. [9] put forward ICFFCM algorithm, which does not use the traditional Euclidean distance to measure the

similarity, but uses fuzzy membership function to define the similarity measurement from pixel to cluster center.

In this paper, we propose a novel extreme filtering fuzzy c-means with neighborhood constraints (EFFCM\_N) clustering segmentation method which is robust to noise. We utilize image extremum filtering operation and construction of segmentation objective function with neighborhood information constraints. The analysis and experimental results show that the proposed method is obviously superior to other FCM methods, and the accuracy is higher.

The rest of this paper is organized as follows: In Section 2, we introduce traditional FCM method firstly, then we review another two improved fuzzy C-means algorithms. Section 3 discusses our algorithm in detail. Section 4 presents and analyzes the experimental results. A conclusion is made in section 5.

## 2. Related Work

### 2.1 Traditional Fuzzy C-means

Given a dataset  $X = \{x_1, x_2, \dots, x_n\}$  where each object  $x_j$ ,  $j = 1, 2, \dots, n$  is an  $l$  dimensional vector. FCM divides objects or patterns under test into  $c$  clusters through an iterative minimization process. The objective function of FCM is defined as:

$$J^{FCM}(U, V) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m \|x_j - v_i\|_2^2 \quad (1)$$

Here,  $U = \{u_{ij}\}, i = 1 \dots c, j = 1 \dots n$  represents the membership degree division matrix, and shall meet the following requirements:  $\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n$ .  $V = \{v_1, v_2, \dots, v_c\}$  is the central variable of the cluster,  $m$  is the fuzzy variable, and usually taking the value of 2. The  $j$  th tested object or mode is expressed as:  $x_j$ , FCM adopts alternative optimization scheme to obtain the best conditions, as shown below:

$$U^{(t+1)} = \arg \min_U J^{FCM}\{U, V^{(t)}\} \quad (2)$$

$$V^{(t+1)} = \arg \min_V J^{FCM}\{U^{(t+1)}, V\} \quad (3)$$

where  $t$  represents the iteration step, and FCM usually randomly initializes  $U^{(0)}$  and  $V^{(0)}$ , and then updates  $U$  and  $V$  alternately until convergence. By using the Lagrange multiplier theorem, it is easy to obtain the calculation of the membership degree partition matrix and the cluster center vector. The calculation formula is as follows:

$$u_{ij}^{(t+1)} = \left( \sum_{k=1}^c \left( \frac{\|x_j - v_i^{(t)}\|_2^2}{\|x_j - v_k^{(t)}\|_2^2} \right)^{\frac{1}{m-1}} \right)^{-1} \quad (4)$$

$$v_i^{(t+1)} = \frac{\sum_{j=1}^n (u_{ij}^{(t+1)})^m x_j}{\sum_{j=1}^n (u_{ij}^{(t+1)})^m} \quad (5)$$

FCM is valid for noiseless data, however, real image data usually contains artifacts such as noise and outliers. The reason why FCM is noise sensitive is that it does not consider spatial information.

## 2.2 Improved FCM

In order to improve the performance of traditional fuzzy C-means clustering algorithm, an improved fuzzy C-means method with bias field data, called bias corrected fuzzy C-means method (BCFCM), is proposed. The objective function is defined as:

$$J^{(BCFCM)}(U, V, \beta) = \sum_{i=1}^c \sum_{j=1}^N u_{ij}^m \|x_j - \beta_j - v_i\|_2^2 + \frac{\alpha}{N_R} \sum_{i=1}^c \sum_{j=1}^N u_{ij}^m \left( \sum_{r \in N_j} \|x_r - \beta_r - v_i\|_2^2 \right) \quad (6)$$

where  $\alpha$  is the smoothing parameter,  $N_R$  is the cardinality of the local window,  $\beta_j$  and  $\beta_r$  are the estimated additive bias fields of pixel  $j$  and its adjacent pixels  $r$ , respectively. Other symbols have the same meaning as FCM. This method is used for deviation correction and clustering. However, the sparsity constraint not imposed on  $j$  usually makes its estimation unstable and unreasonable.

## 2.3 FCM of Local Information

On the basis of predecessors, someone introduced a new fuzzy factor  $G_{ij}$  into the objective function of the FCM algorithm, which is defined as:

$$G_{ij} = \sum_{\substack{r \in N_j \\ r \neq j}} \frac{1}{1 + d_{jr}} (1 - u_{ir})^m \|x_r - v_i\|_2^2 \quad (7)$$

where  $N_j$  is the local window with the center of  $j$ ,  $r$  belongs to the neighborhood pixel of  $j$ ,  $d_{jr}$  is the Euclidean distance between  $j$  and  $r$ , and  $u_{ir}$  represents the member of  $r$  belonging to the  $i$ th cluster.

Although the fuzzy C-means based on local information is effective for images damaged by noise or outliers, even though adjacent membership  $u_{ir}$  can interact with  $u_{ij}$ , the fuzzy factor  $G_{ij}$  is considered constant in each iteration step of the frame. In relevant literature, it has been said that the iterative updating formula in the fuzzy C-means clustering algorithm based on local information can not actually minimize the objective function.

## 3. Proposed Method

### 3.1 Image Extremum Filtering

The derivative of local image contains abundant structural information, which has great potential in texture analysis. Inspired by relevant literature, in this section, we use the maneuverability of

Gaussian derivative filters (including first-order and second-order) to calculate the extreme (including maximum and minimum) response in the local image scale space. These operations are called "extreme filtering" (EF) operations. We introduce the following motives for extreme value filtering: first, we can capture a lot of useful information in the first and second order differential structures within a certain range. Second, we can extract local features that are completely rotation invariant. Third, we can make our method more efficient.

According to the theory of controllable filters, the linear combination of multiple basic filters can be used to synthesize any direction of the first or second derivative of Gauss. We adopt two-dimensional circularly symmetric Gaussian function [10], which is defined as:

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (8)$$

where  $\sigma$  is the standard deviation or scale, and the first and second derivatives of  $\theta$  in any direction are:

$$G_1^\theta = \cos(\theta)G_x + \sin(\theta)G_y \quad (9)$$

$$G_2^\theta = \cos^2(\theta)G_{xx} - \sin(2\theta)G_{xy} + \sin^2(\theta)G_{yy} \quad (10)$$

For the obtained extreme response, we construct a distinctive transform feature set to characterize the local texture structure and its correlation. Note that the set we construct is the image set after extreme filtering, which is defined as:

$$\eta = I_{2\max}^\theta - I_{2\min}^\theta + I_{1\max}^\theta \quad (11)$$

FCM is sensitive to noise, because the distribution characteristics of data are always affected by noise damage, which will lead to two problems: first, the results obtained by FCM algorithm are not good for noisy image segmentation. On the other hand, for images damaged by noise, the number of iterations of FCM is larger than that of images not damaged by noise. Therefore, in the next section, before applying clustering, we introduce the extreme filtered image into FCM algorithm to optimize the distribution characteristics of data.

### 3.2 FCM Clustering Segmentation of Image based on Extremum Filtering

Based on the extreme filtered images obtained above, we propose an extreme filtering fuzzy C-means (EFFCM) clustering segmentation algorithm. The clustering of EFFCM is performed on the gray histogram, so the objective function can be written as:

$$J_m = \sum_{l=1}^q \sum_{k=1}^c \xi_l u_{kl}^m \|\eta_l - v_k\|^2 \quad (12)$$

where  $\eta$  is the extreme filtered image (Formula 11),  $u_{kl}$  represents the membership function of the gray value  $l$  relative to the cluster center  $k$ . And that,

$$\sum_{l=1}^q \xi_l = N \quad (13)$$

where  $\eta_l$  is the gray level image after extreme filtering,  $1 \leq l \leq q$ ,  $q$  represents the number of gray levels contained in  $\eta$ , and its value is usually much smaller than  $N$ .

### 3.3 FCM Clustering Segmentation with Neighborhood Information Constraints

In contrast to the C-V model,

As we all know, pixels in an image are not isolated. There may be a connection between each pixel and its adjacent pixels. In addition, the distribution of different individuals in many images is complex or even overlapping. Moreover, most image quality will be degraded due to noise. Therefore, the local neighborhood information must be combined into the clustering process to further suppress the impact of noise, which will further improve the segmentation accuracy. In order to further improve the clustering performance of images, we introduce neighborhood information into EFFCM, and propose extreme filtering fuzzy C-means clustering segmentation algorithm with neighborhood information constraints (EFFCM\_N). The objective function of EFFCM\_N is defined as follows:

$$J_m = \sum_{l=1}^n \sum_{k=1}^c \xi_l u_{kl}^m \|\eta_l - v_k\|^2 + \lambda \sum_{l=1}^n \sum_{k=1}^c u_{kl}^m \sum_{j \in N_l} \frac{(\eta_j - v_k)^2}{\text{card}(N_l)} \quad (14)$$

Where  $n$  is the total number of pixels after extreme filtering,  $N_l$  represents a group of neighborhood pixels falling into the window around pixel  $i$ ,  $\text{card}(N_l)$  is the cardinal number of  $N_l$ , and other parameters are the same as those described above. The main process of EFFCM\_N segmentation is similar to the FCM algorithm described above. It also uses the Lagrangian multiplier theorem to obtain the value of the membership matrix and the cluster center vector, so as to obtain the final cluster segmentation result.

**Table 1.** The average number of pixels that are incorrectly segmented corresponds to different  $\lambda$

Take the value of $\lambda$	Average number of mis-segmented pixels
$\lambda=0$	3748
$\lambda=0.1$	3725
$\lambda=0.2$	3685
$\lambda=0.3$	3657
$\lambda=0.4$	3633
$\lambda=0.5$	3633
$\lambda=0.6$	3632
$\lambda=0.7$	3630
$\lambda=0.8$	3631

When adding local neighborhood information items to the objective function, we will consider the local information of each pixel in the whole clustering process. In fact, the neighborhood term can be

regarded as the regularization term that makes the segmentation homogeneous. Therefore, the influence of noise can be further suppressed.

The  $\lambda$  in formula 14 controls the effect of local neighborhood information items. In order to study the influence of the value of  $\lambda$  on the experimental results of our EFFCM\_N algorithm, we choose different values of  $\lambda$  from 0 to 1 for experiments, where the growth step is 0.1. Table 1 shows the average number of natural image missegmented pixels under different  $\lambda$  values. It can be seen from Table 1 that the average quantity first decreases with the increase of  $\lambda$  value, but when  $\lambda=0.4$ , the quantity value tends to be stable and does not change any more. Therefore, this shows that the local neighborhood information is effective, and the results are insensitive to  $\lambda$  within a certain range. Therefore, for the natural image segmentation of EFFCM\_N algorithm, set the value of  $\lambda$  to 0.4.

## 4. Experiments

In this part, in order to evaluate the effectiveness and efficiency of our proposed EFFCM\_N algorithm, we tested synthetic noise images and natural color images in experiments. Our algorithm is compared with several latest clustering algorithms, including FCM [7] and FCM\_S1 [11], FCM\_S2 [11], EnFCM [12], FGFCM [13], FLICM [14], KWFLICM [15] and NWFCM [16] algorithms. These algorithms have different advantages, among which FCM, FCM\_S1, FCM\_S2, EnFCM, FGFCM and NDFCM have lower computational time complexity. FLICM, KWFLICM and NWFCM have strong noise elimination capabilities. In addition, FLICM and KWFLICM do not need to set parameter values. They are both fuzzy clustering segmentation methods combining local and non local information. Therefore, we choose these algorithms as comparison methods.

The experimental results of various algorithms are evaluated and analyzed qualitatively and quantitatively. Many datasets provide accurate segmentation results manually. We use two common criteria (including dice coefficient (DC) and segmentation accuracy (SA)) to evaluate the segmentation results quantitatively. DC is defined as:

$$DC(R, T) = \frac{2|R \cap T|}{|R| + |T|} \quad (15)$$

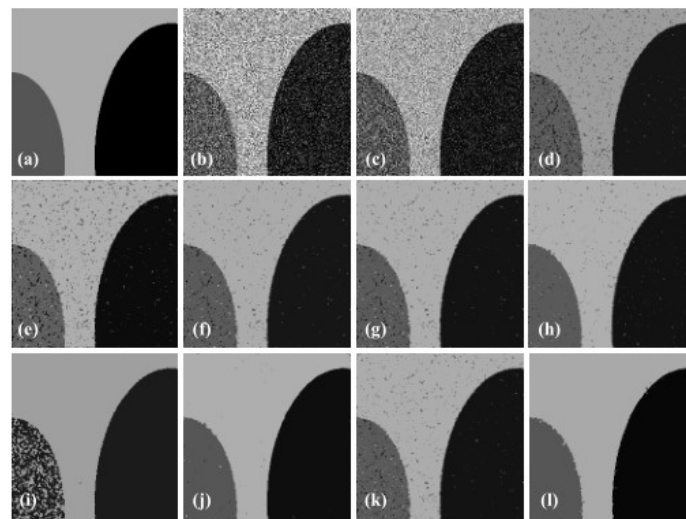
R and T represent the segmentation result and the corresponding artificial accurate segmentation image respectively. The larger the DC value, the more accurate the segmentation result of the algorithm. In addition, the segmentation accuracy SA is defined as:

$$SA(R, T) = \frac{\sum_{i=1}^c \frac{|R_i \cap T_i|}{|T_i|}}{\sum_{j=1}^c 1} \quad (16)$$

where  $c$  is the number of clusters, and SA is the ratio of the number of correctly classified pixels to the total number of pixels. Therefore, SA can measure the accuracy of the overall segmentation. The higher the SA value, the more accurate the result.

### 4.1 Experimental Results of Composite Images

In this section, we used a composite image of size in our experiment. The image includes three categories (three intensity values are 0, 85 and 170 respectively). The images are damaged by Gaussian, salt and pepper and uniform noise respectively, and the damaged images are used to test the EFFCM proposed above. The efficiency and robustness of N algorithm. Figure 1 shows the segmentation results obtained on this image by comparing different algorithms.



**Figure 1.** Comparison of segmentation results on the noisy image among different methods. (a) Original image. (b) noisy image. (c) FCM result. (d) FCM\_S1 result. (e) FCM\_S2 result. (f) EnFCM result. (g) FGFCM result. (h) FLICM result. (i) NWFCM result. (j) KWFLICM result. (k) NDFCM result. (l) EFFCM\_N result

It can be seen from Table 2 and Table 3 that the segmentation accuracy of EFFCM\_N is always higher than that of other algorithms for composite images containing different noises. Obviously, compared with other algorithms, EFFCM\_N is much more robust to different noises.

**Table 2.** SA value of experimental results of different algorithms on different noise images

Noise	FCM	FCM_S1	EnFCM	FLICM	NDFCM	EFFCM_N
3% Gaussian noise	0.7370	0.9847	0.9886	0.9910	0.9881	0.9968
10% Gaussian noise	0.5967	0.8053	0.8817	0.9085	0.8749	0.9945
10% pepper noise	0.9428	0.9485	0.9570	0.9337	0.9860	0.9992
20% pepper noise	0.8342	0.8831	0.8684	0.8201	0.9517	0.9965
10% uniform noise	0.9364	0.9705	0.9808	0.9653	0.9881	0.9987
20% uniform noise	0.8714	0.9309	0.9482	0.9022	0.9643	0.9975

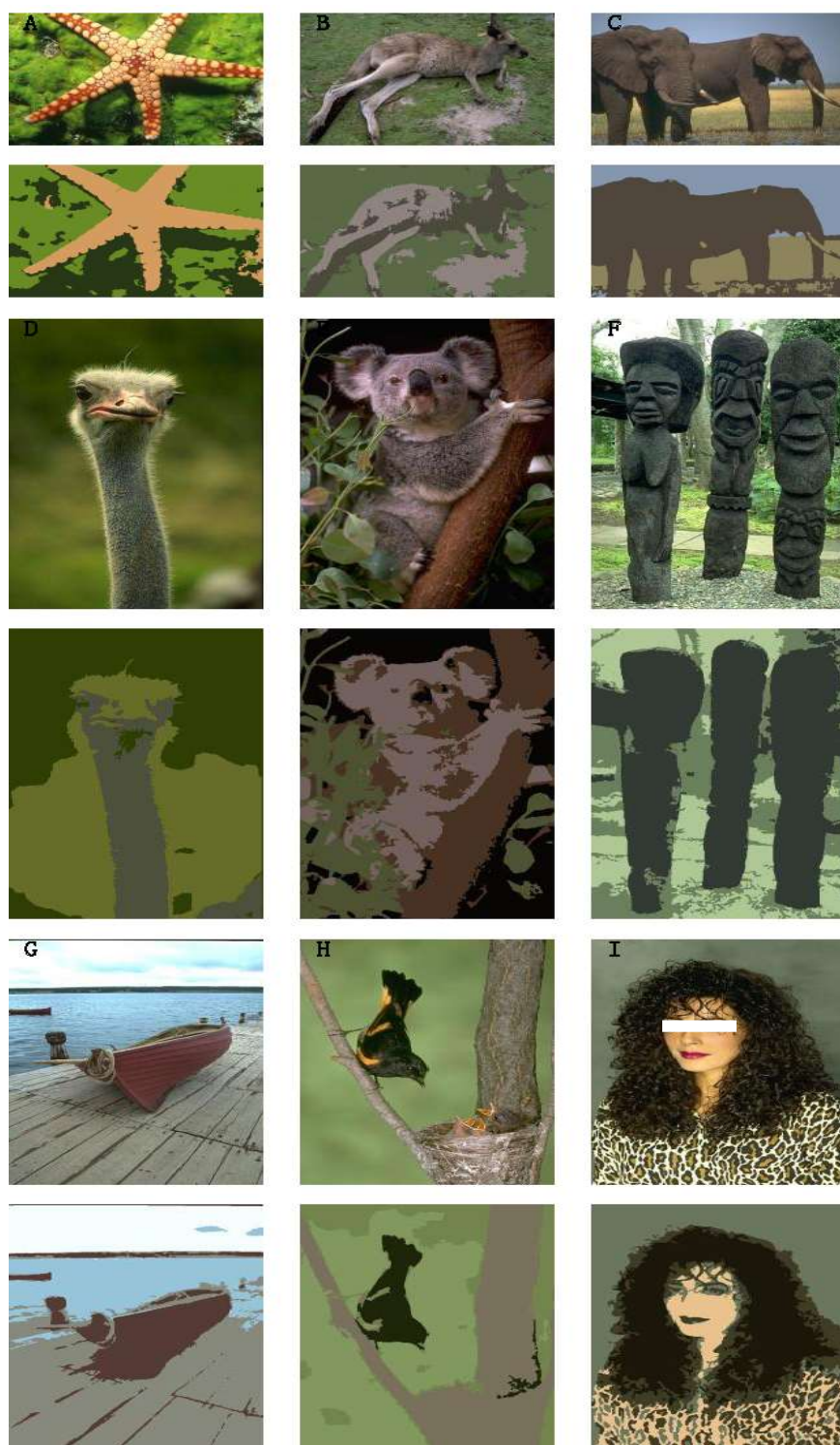
**Table 3.** DC value of experimental results of different algorithms on different noise images

Noise	FCM	FCM_S1	EnFCM	FLICM	NDFCM	EFFCM_N
3% Gaussian noise	0.7234	0.8657	0.9643	0.9234	0.9456	0.9934
10% Gaussian noise	0.5342	0.8123	0.8829	0.8970	0.8908	0.9967
10% pepper noise	0.8432	0.9346	0.8960	0.9234	0.9256	0.9979
20% pepper noise	0.7681	0.8964	0.8234	0.8458	0.9297	0.9954
10% uniform noise	0.8945	0.9345	0.9406	0.9345	0.9239	0.9989
20% uniform noise	0.9067	0.9392	0.9604	0.9012	0.9689	0.9923



## 4.2 Experimental Results on Natural Images

In this section, we will prove the advantages of our EFFCM\_N algorithm in natural images. To demonstrate the advantages of EFFCM\_N algorithm, we conducted experiments on natural images on BSDS500 dataset. Figure 2 shows the 9 natural images we used for experiments and the results of EFFCM\_N method. Table 4 and Table 5 respectively show the DC and SA values of the segmentation results of the nine images in Figure 2. It can be seen from the two tables that the segmentation accuracy of our EFFCM\_N algorithm is higher than that of other algorithms.



**Figure 2.** Nine natural color images A-I and segmentation results



**Table 4.** SA values of experimental results of various algorithms on natural images

image	FCM	FCM_S1	EnFCM	FLICM	NDFCM	EFFCM_N
image A	0.6745	0.8432	0.9523	0.9103	0.9219	0.9923
image B	0.5678	0.8145	0.8219	0.8657	0.9069	0.9968
image C	0.8278	0.9210	0.8642	0.9103	0.9137	0.9977
image D	0.7679	0.8671	0.8987	0.8349	0.9395	0.9949
image E	0.7801	0.9453	0.8569	0.9109	0.9296	0.9981
image F	0.9102	0.9218	0.9219	0.8910	0.9561	0.9957
image G	0.8705	0.8934	0.8321	0.8764	0.9103	0.9920
image H	0.8854	0.8659	0.8734	0.8542	0.9219	0.9919
image I	0.4328	0.9201	0.8563	0.8570	0.9436	0.9924

**Table 5.** DC values of experimental results of various algorithms on natural images

image	FCM	FCM_S1	EnFCM	FLICM	NDFCM	EFFCM_N
image A	0.6853	0.8345	0.9458	0.9104	0.9359	0.9935
image B	0.4598	0.8196	0.8458	0.8609	0.8503	0.9918
image C	0.7860	0.8962	0.9231	0.8941	0.9120	0.9971
image D	0.7621	0.9034	0.8642	0.8719	0.9359	0.9968
image E	0.8128	0.9290	0.9215	0.9250	0.9210	0.9919
image F	0.8904	0.9128	0.9549	0.9360	0.9574	0.9926
image G	0.7821	0.8603	0.8971	0.8703	0.9018	0.9946
image H	0.6853	0.8319	0.8621	0.8932	0.9463	0.9931
image I	0.5430	0.9023	0.9310	0.9205	0.9852	0.9940

## 5. Conclusion

In this paper, we propose an extreme filtering fuzzy C-means clustering segmentation algorithm with local neighborhood restrictions, namely EFFCM\_N algorithm. EFFCM\_N algorithm does not directly use the original image for experiments. It first processes the image with extreme filtering, and then uses it for clustering experiments. The extreme value filtering operation redistributes the image data, which increases the calculation speed and is more robust to noise. Then, we substitute the filtered image histogram into the objective function of FCM to achieve image segmentation. Then, in order to further improve the segmentation performance of the clustering algorithm, we add local neighborhood information to constrain the clustering process. That is our EFFCM\_N algorithm. This algorithm combines multi-scale extreme value filtering and local spatial information, and their advantages are mutually enhanced. Compared with the original fuzzy C-means clustering algorithm, the segmentation performance is improved.

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