

Research on Coordination of Repurchase and Revenue-sharing Contract Considering Government Subsidies

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Abstract

Based on the supplier repurchase contract and the revenue-sharing contract, this paper establishes a supplier decision-making model under the government's differential subsidy of supplier repurchase costs, and explores the influence of the government's differential subsidy on the wholesale price of suppliers and the order quantity of retailers. Studies have shown that under the condition of government subsidies, the optimal order quantity of retailers in the supply chain under decentralized decision-making has increased significantly; under centralized decision-making, the overall expected profit of the supply chain has increased significantly. Under the revenue-sharing contract, only when the government subsidy ratio is within a certain range, the retailer will have the optimal order quantity, so as to achieve the coordination of the supply chain.

Keywords

Repurchase; Government Subsidy; Revenue-sharing Contract.

1. Introduction

As of the end of 2018, the number of small and medium-sized enterprises in China has exceeded 30 million, and the number of individual industrial and commercial households has exceeded 70 million, contributing more than 50% of the country's tax revenue, more than 60% of GDP, more than 70% of technological innovation and 80% More than% of the labor force is employed. The contribution rate of small and medium-sized enterprises to China's economic development continues to create historical highs. However, financing difficulties have become a problem that restricts the development of small and medium-sized enterprises in my country, and it has also caused the small and medium-sized enterprises to be unable to implement the optimal ordering decision, and the enterprises in the supply chain cannot make the optimal decision. As a relatively new model of supply chain finance, supplier repurchase guarantee financing can control the guarantee risks of core enterprises and effectively solve the problems of supply chain risk sharing and benefit enhancement. In reality, not all suppliers have the ability to guarantee the repurchase of downstream retailers. Therefore, this article discusses the impact of government repurchase subsidies on the decision-making of enterprises in the supply chain. In the previous literature, many scholars have conducted research on repurchase guarantees and revenue-sharing contracts. Padmanabhan[1] research shows that repurchase can increase the profit and competitiveness of sellers and suppliers at the same time; repurchase activities are more effective when the demand is uncertain and the supplier's cost is low. Ma Hui[2] designed a complete repurchase contract based on commercial credit, and established a repurchase and revenue sharing contract model on this basis. Yi Xuehui[3] believes that the repurchase contract in the supply chain contract is a kind of option contract, which can share risks and improve performance in supply chain coordination. Cai Xin[4] believe that only when the parameters meet a specific relationship, the repurchase contract can achieve supply chain coordination under inventory pledge financing. This kind of literature studies various aspects

such as the optimal order quantity and the optimal pricing under the repurchase contract, but they seldom consider the influence of government subsidies on the repurchase guarantee subsidy on the repurchase contract. Regarding the research on revenue-sharing contracts, Caehon, Lariviere MA[5] have studied the advantages and limitations of revenue-sharing contracts. Yu Liping[6] established a revenue sharing contract coordination model based on commercial credit under random demand, analyzed the role of revenue sharing mechanism under deferred payment in realizing the reasonable distribution of revenue and improving the overall operational performance of the supply chain, and derived the realization Optimal contract parameters for decentralized supply chain coordination.

In recent years, research on the development of supply chains by government subsidies has also been extremely popular. Cao Yu[7] et al. explored the impact of government subsidy strategies on green efforts and pricing decisions in the supply chain when the green efforts of manufacturers and retailers are considered at the same time. Research shows that government subsidies can stimulate the green efforts of manufacturers and retailers. Based on green preference, Wen Xingqi[8] et al. established three manufacturer subsidy game models based on greenness, production costs, and R&D costs, and compared the effects of the same government subsidy expenditures. Mo Zan[9] et al. believe that compared with the situation of no government subsidies, government subsidies can pass the overall performance of the supply chain and social welfare, but excessive subsidies will lead to a decline in social welfare. In previous studies, scholars have mainly considered government subsidies for manufacturers' innovation and R&D costs and consumer subsidies for purchasing green products. Few scholars have noticed government subsidies for repurchase costs.

Therefore, the article constructs a secondary supply chain of a single supplier and a single retailer.

In the case that the supplier is the core enterprise fully repurchases the part of the retailer whose order quantity exceeds the demand, consider whether there is a government subsidy to the supplier's repurchase guarantee fee. After that, we adopted a revenue-sharing contract to study the impact of government subsidies on the retailer's optimal order quantity and the supplier's optimal wholesale price.

2. Model and Symbol Definition

Consider a secondary supply chain consisting of a single retailer and a single supplier. The supplier supplies the retailer at the wholesale price, and the retail price p of the product remains unchanged during the sales cycle. The retailer has limited initial funds. In order to order the required product q to obtain more profits, the retailer uses pq as collateral to apply for a loan from the bank. The bank provides a loan λpq to the retailer with a certain pledge rate λ and interest rate r . The market demand is random during the sales period. The retailer sells the ordered product until the product is sold out or the sales period ends. If the product is sold out, because the consumer demand cannot be met, part of the loss of the consumer's shortage will be borne by the retailer and part by the supplier. If there is still a surplus of the product at the end of the sales period, the supplier as the core enterprise of the supply chain promises to be at the end of the sales period According to a certain price b and repurchase products not sold by the retailer.

Assumptions:

- (1) Both the supplier and the retailer are risk-neutral, and the information between the two is complete;
- (2) We assume that the range of government subsidy ratio is: $\theta \in (0,1)$.
- (3) The retailer's own funds are far less than the optimal order quantity.

(4) The distribution function of D is $F(x)$, the density function is $f(x)$, $F(x)$ is differentiable and strictly increasing, $F(0) = 0$, the expected value of D is $\mu = E(D)$.

Table 1. Symbol definition

Definition	Explanation
c	cost
g_s	Loss of stock out of supplier unit
g_r	Retailer unit loss of stock out
v	Unit product residual value
p	Retailer's unit product price
w	wholesale price
b	Repurchase price
q	Retailer order quantity
D	Random market demand
θ	Government subsidy ratio
r	Bank loan interest rate
λ	Inventory pledge rate set by the bank
φ	Revenue sharing parameters
π_s	Supplier profit function
$E(\pi_s)$	Supplier's expected profit

3. Decisions in the Supply Chain Based on Repurchase Contracts

3.1. Decision-making under no-government repurchase guarantee subsidy

First, we discuss the difference between the optimal wholesale price and the optimal order quantity in the supply chain when there is no government subsidy for the repurchase guarantee fee, and the supply chain adopts decentralized decision-making and centralized decision-making.

3.1.1. Decentralized decision-making situation of no-government repo guarantee subsidies

Regardless of government subsidies, the supplier, as the core enterprise in the supply chain, is the leading enterprise in the supply chain, and the retailer is the follower. The supplier determines its own repurchase price and then the retailer determines its optimal order quantity. When the retailer's order quantity is greater than the market demand, the supplier will repurchase at the price of b ; when the order quantity is less than the market demand, the supplier will incur stock-out costs. The supplier's profit function is:

$$\begin{cases} \pi_s = (w - c)q - (b - v)q - D & D \leq q \\ \pi_s = (w - c)q - g_s(D - q) & D > q \end{cases} \quad (1)$$

The supplier's expected profit is:

$$\begin{aligned} E(\pi_s) = & \int_0^q (w - c)q f(x) dx - \int_0^q (b - v)(q - x) f(x) dx \\ & + \int_q^\infty (w - c)q f(x) dx + \int_q^\infty g_s (x - q) f(x) dx \end{aligned} \quad (2)$$

After algebraic calculation, the supplier's expected profit is:

$$E(\pi_s) = (w - c + g_s) q - (b + g_s - v) \int_0^q (q - x) f(x) dx - g_s \mu \quad (3)$$

When the retailer's order quantity exceeds the market demand, the supplier's complete repurchase strategy will reduce the retailer's bankruptcy risk. We assume that the retailer's own funds are not enough to order the optimal quantity, and the retailer's loan amount is λpq , then the retail profit function is:

$$\begin{cases} \pi_r = (p - w)q - g_r(D - q) + \lambda pq - \lambda pq(1 + r) & D > q \\ \pi_r = pD - wq + b(q - D) + \lambda pq - \lambda pq(1 + r) & D \leq q \end{cases} \quad (4)$$

The expected profit of the retailer is:

$$E(\pi_r) = ((p + g_r - w - \lambda pr)q - (p + g_r - b) \int_0^q (q - x)f(x)dx) - g_r\mu \quad (5)$$

In equation (5), since $\frac{\partial^2 E(\pi_r)}{\partial q^2} < 0$, $E(\pi_r)$ is a strictly concave function, so the necessary and sufficient condition for q to be its optimal solution is q Satisfy its first-order optimality condition

$$\begin{aligned} \frac{\partial E(\pi_r)}{\partial q} &= (p - w + g_r - \lambda pr) - (p + g_r - b)F(q) - g_r\mu \\ \frac{\partial^2 E(\pi_r)}{\partial q^2} &= -(p + g_r - b)f(q) < 0 \end{aligned}$$

$$F(q^1) = \frac{p + g_r - w - \lambda pr}{p + g_r - b} \quad (6)$$

$$q^1 = F^{-1}\left(\frac{p + g_r - w - \lambda pr}{p + g_r - b}\right) \quad (7)$$

It can be seen from equation (7) that the retailer's order quantity will decrease as the wholesale price w increases. Let $\frac{\partial E(\pi_s)}{\partial w} = 0$, get the supplier's best wholesale price w^1 :

$$\frac{\partial \left\{ \psi(w^1) + \int_0^{\psi(w^1)} [\psi(w^1) - x]f(q^1)dx \right\}}{\partial w^1} = \frac{\psi(w^1)}{b + g_s - v} \quad (8)$$

$$q^1 = \psi(w^1), F(q^1) = \frac{p + g_r - w - \lambda pr}{p + g_r - b}$$

3.1.2. The centralized decision-making situation of the non-government repurchase guarantee subsidy

The expected profit of the two-level supply chain system under the full repurchase strategy is:

$$E(\pi) = (p + g - c - \lambda qr)q - (p + g - v) \int_0^q F(x)dx - g\mu \quad (9)$$

The optimal order quantity q^2 of the supply chain system under the full repurchase strategy is given below:

In equation (9), since $\frac{\partial^2 \pi}{\partial q^2} < 0$, it is a strictly concave function, so the necessary and sufficient condition for q to be its optimal solution is that q satisfies its first-order maximum Superiority condition.

$$\frac{\partial E(\pi)}{\partial q} = (p + g - \lambda qr - c) - (p + g - v)F(x) = 0 \quad (10)$$

$$F(q^*) = \frac{p + g - c - \lambda qr}{p + g - v} \quad (11)$$

3.2. Decisions under government subsidy repurchase guarantees

The ratio of government guarantees for supplier repurchase is θ , and the supplier's repurchase price is b . On the basis of government subsidies for repurchase guarantee costs, the supplier's repurchase fee is $(1 - \theta)b(q - x)^+$

3.2.1. Decentralized decision-making situation with government repurchase guarantee subsidies

The expected profit function of the supplier under the government repurchase guarantee subsidy is:

$$E(\pi_s) = (w + g_s - c)q - [(1 - \theta)b + g_s - v] \int_0^q (q - x)f(x)dx - g_s\mu \quad (12)$$

Similarly, we can calculate the retailer's expected profit function as:

$$E(\pi_r) = ((p + g_r - w - \lambda pr)q - (p + g_r - b) \int_0^q (q - x)f(x)dx) - g_r\mu \quad (13)$$

In equation (14), since $\frac{\partial^2 E(\pi_r)}{\partial q^2} < 0$, $E(\pi_r)$ is a strictly concave function, so the necessary and sufficient condition for q to be its optimal solution is q Satisfy its first-order optimality condition.

$$\begin{aligned}\frac{\partial E(\pi_r)}{\partial q} &= (p - w + g_r - \lambda pr)q - (p + g_r - b)F(q) - g_r\mu \\ \frac{\partial^2 E(\pi_r)}{\partial q^2} &= -(p + g_r - b)f(q) < 0 \\ F(q^2) &= \frac{p + g_r - w - \lambda pr}{p + g_r - b}\end{aligned}\quad (14)$$

$$q^2 = F^{-1}\left(\frac{p + g_r - w - \lambda pr}{p + g_r - b}\right) \quad (15)$$

It can be seen from equation (16) that the retailer's order quantity will decrease as the wholesale price increases. Let $\frac{\partial E(\pi_s)}{\partial w} = 0$, get the supplier's best wholesale price w^2 :

$$\begin{aligned}\frac{\partial \left\{ \psi(w^2) + \int_0^{\psi(w^2)} [\psi(w^2) - x] f(q^2) dx \right\}}{\partial w^2} &= \frac{\psi(w^2)}{(1-\theta)b + g_s - v} \\ q^2 &= \psi(w^2) \quad F(q^2) = \frac{p + g_r - w - \lambda pr}{p + g_r - b}\end{aligned}\quad (16)$$

Comparing formula (8) and formula (16), we find that under the decentralized decision with government repurchase guarantee subsidies, the wholesale price of suppliers decreases, because the retailer's optimal order quantity is a function of the wholesale price, and Increase with the decrease of wholesale price. Therefore, in contrast to the decentralized decision with or without government repurchase guarantee subsidies, when there are government repurchase subsidies, the retailer's optimal order quantity increases. That is $q^1 < q^2$.

3.2.2. Centralized decision-making situation with government repurchase guarantee subsidies

The expected profit function of the supplier under the government repurchase guarantee subsidy is:

$$E(\pi_s) = (w + g_s - c)q - [(1 - \theta)b + g_s - v] \int_0^q (q - x)f(x)dx - g_s\mu \quad (17)$$

The retailer's expected profit function is:

$$E(\pi_r) = (p + g_r - w - \lambda pr)q - (p + g_r - b) \int_0^q (q - x)f(x)dx - g_r\mu \quad (18)$$

The expected profit function of the entire supply chain is as follows:

$$E(\pi) = (p + g - c - \lambda pr)q - (p + g - v - \theta b) \int_0^q (q - x)f(x)dx - g\mu \quad (19)$$

The optimal order quantity q^* of the supply chain system under the complete repurchase strategy is given below:

In equation (19), $\frac{\partial^2 \pi}{\partial q^2} < 0$, it is a strictly concave function, so the necessary and sufficient component for q is its optimal solution is that q satisfies its first order optimality condition.

$$\frac{\partial E(\pi)}{\partial q} = (p + g - \lambda pr - c) - (p + g - v - \theta b)F(q) = 0 \quad (20)$$

$$F(q^{**}) = \frac{p + g - c - \lambda pr}{p + g - v - \theta b} \quad (21)$$

By comparing formula (11) and formula (21), we can find that $q^{**} > q^*$, so we can reach the conclusion that the retailer's cost can be increased when the government subsidizes the cost of repurchase guarantees. order amount. We bring formula (11) and formula (22) into formula (9) and formula (20) respectively, and we find that the overall expected target profit of the supply chain under the government subsidized repurchase guarantee fee is higher.

Under the decentralized decision-making, the expression of the retailer's optimal order quantity is brought into the supplier's expected profit function. In the case of government subsidies, the supplier will lower the wholesale price, thus prompting the retailer to increase the order quantity. Under centralized decision-making, we see that the optimal order quantity with government subsidies is higher than the optimal order quantity without government subsidies. This is because, under government subsidies, suppliers' repurchase efforts have been strengthened, giving retailers the confidence to increase their order volume.

Table 2. Comparison in two cases

	No government subsidy		With government subsidy	
	Decentralized decision-making	Centralized decision-making	Decentralized decision-making	Centralized decision-making
Optimal order quantity	$F(q^1) = \frac{p+g_r-w-\lambda pr}{p+g_r-b}$	$F(q^*) = \frac{p+g-c-\lambda qr}{p+g-v}$	$F(q^2) = \frac{p+g_r-w-\lambda pr}{p+g_r-b}$	$F(q^{**}) = \frac{p+g-c-\lambda pr}{p+g-v-\theta b}$
Optimal wholesale price	$\frac{\psi(w^1)}{b+g_s-v}$		$\frac{\psi(w^2)}{(1-\theta)b+g_s-v}$	

4. Supply Chain Decision-making with Government Subsidies under the Revenue-sharing Contract

In order to achieve the goal of win-win and mutual benefit and promote effective cooperation between the two parties, the revenue sharing parameter φ ($0 < \varphi < 1$) is introduced to determine the distribution of all expected revenues between the supplier and the retailer, and φ represents the profit shared by the retailer Ratio, the retailer's expected profit function is

$$E(\pi_r) = \varphi R(q) + b \int_0^q (q-x)f(x)dx - wq - \lambda qpr - g_r \int_q^\infty (x-q)f(x)dx \quad (22)$$

$R(q) = pq - (p-v) \int_0^q (q-x)f(x)dx + \theta b \int_0^q (q-x)f(x)dx$ is the expected total revenue of the supply chain system.

The second term represents the retailer's revenue when the supplier repurchases the remaining products, the third term represents the supplier's cost, the fourth term represents the retailer's loan interest, and the fifth term represents the retailer's loss of stock-out.

Similarly, we can get the supplier's expected profit function as

$$E(\pi_s) = (1-\varphi) R(q) + wq - b \int_0^q (q-x)f(x)dx - cq - g_r \int_q^\infty (x-q)f(x)dx \quad (23)$$

The first item indicates the proportion of revenue that the supplier receives, the second item indicates the income from selling the product to the retailer, the third item indicates the capital investment in repurchasing the remaining products, the fourth item indicates the total cost of the supplier, and the fifth item represents the supplier's loss of stock out.

In a decentralized supply chain, suppliers and retailers make independent decisions, which may affect the performance of the entire supply chain and cause the problem of "double marginalization". The purpose of designing a supply chain complete repurchase and revenue sharing joint contract is to seek integrated supply chain operational performance. In the integrated supply chain, suppliers and retailers are vertically integrated enterprises, adopting centralized control to determine the optimal order quantity and wholesale price that maximize the total profit of the supply chain.

The expected profit function of the integrated supply chain system is

$$E(\pi) = R(q) - cq - \lambda qpr - g \int_q^\infty (x-q)f(x)dx \quad (24)$$

At this time, from the first-order condition of equation (19), the order quantity q^∇ that maximizes the total profit of the supply chain system can satisfy the following equation q^∇

$$F(q^\nabla) = \frac{p - \lambda qr - c + g}{p - v - \theta b + g} \quad (25)$$

From this formula, we can see that the retailer's optimal order quantity is positively correlated with the government subsidy ratio and repurchase price. That is to say, the higher the government's repurchase subsidy ratio, the greater the retailer's optimal order quantity; The higher the repurchase price, the greater the retailer's optimal order quantity.

Next, by maximizing the total profit of the supply chain system and also maximizing the profit of retailers and suppliers in the decentralized supply chain, we can realize the coordination of the decentralized supply chain and achieve the operational performance of the integrated supply chain. According to formula (23) and formula (24), we can obtain the optimal order quantity of the decentralized decision-making retailer:

Retailer's expected return function:

$$E(\pi_r) = \varphi \left[pq - (p - v) \int_0^q (q - x)f(x)dx + \theta b \int_0^q (q - x)f(x)dx \right] + b \int_0^q (q - x)f(x)dx - wq - \lambda pqr - g_r \int_q^\infty (x - q)f(x)dx \quad (26)$$

The first-order condition that satisfies the optimal solution is

$$\frac{\partial E(\pi_r)}{\partial q} = \varphi p - \lambda qr - g_r - [\varphi(p - v - \theta b) - b + g]F(q) = 0 \quad (27)$$

$$\frac{\partial^2 E(\pi_r)}{\partial q^2} = -[\varphi(p - v - \theta b) - b + g_r]f(q) \quad (28)$$

Need to meet $\theta < \frac{p\varphi - v\varphi - b + g_r}{b\varphi}$, $\frac{\partial^2 E(\pi_r)}{\partial q^2} < 0$, In this case, we get the retailer's optimal order quantity under decentralized decision-making.

$$F(q^\Delta) = \frac{\varphi p - \lambda pr - g_r}{\varphi(p - v - \theta b) - b + g_r} \quad (29)$$

This shows that the government's subsidies for supplier repurchase are not as much as possible, but should be within a certain range.

We make the optimal order quantity under centralized decision-making and decentralized decision-making equal, and obtain the expression of the sharing parameters of supply chain coordination:

$$\frac{p - \lambda qr - c + g}{p - v - \theta b + g} = \frac{\varphi p - \lambda pr - g_r}{\varphi(p - v - \theta b) - b + g_r} \quad (30)$$

$$\varphi = \frac{(b - g_r)(p - \lambda qr - c + g) - (\lambda qr - g_r)(p - v + \theta b + g)}{(p - v - \theta b)(p - \lambda qr - c + g) - p(p - v - \theta b + g)} \quad (31)$$

5. Conclusion

The article constructs a secondary supply chain of a single supplier and a single retailer. When the supplier is the core enterprise that fully repurchases the part of the retailer whose order quantity exceeds the demand, consider whether there is a government subsidy to the supplier's repurchase guarantee fee. We find that under the decentralized decision-making, when the government subsidizes repurchase, the supplier's wholesale price decreases. Therefore, although the retailer's revenue function remains unchanged, the retailer's optimal order quantity decreases with the decrease of the wholesale price increase. Under centralized decision-making, compared with no government subsidies, the retailer's optimal order quantity increases when there are government subsidies, and the profits of both the supplier and the retailer increase. Later, we introduced a revenue-sharing contract under the centralized decision-making of government subsidies. Research has shown that only when government

subsidies are within a certain range, retailers will have the optimal order quantity, so as to achieve supply chain coordination. This research will help the government to formulate reasonable repurchase guarantee subsidies. At the same time, it will provide reference for enterprises to make relevant decisions. This research can also consider supply chain coordination under the government subsidized wholesale price contract. At the same time, it can also consider government pledges of accounts receivable, order pledges, repurchase guarantee subsidies, and so on.

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