

The Relationship between Investor Sentiment and Stock Market Return Volatility

-- An Applied Analysis based on Directed Acyclic Graphs

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Abstract

In recent years, with the in-depth study of behavioural finance in the stock market, there is growing evidence that investors' cognitive biases and emotional biases are important factors contributing to stock market volatility. For an emerging stock market such as the Chinese stock market, the application of investor sentiment theory can provide an in-depth investigation into the irrational causes of stock market volatility other than fundamental factors, and thus provide an empirical basis for the prevention and mitigation of financial system risks. This paper investigates the contemporaneous and dynamic relationship between stock market volatility and investor sentiment based on the results of a directed acyclic graph analysis. The results of the study show that, in terms of contemporaneous effects, changes in stock market volatility and individual investor sentiment cause changes in institutional investor sentiment. The results of the fixed sample period and recursive predictive variance decomposition based on directed acyclic graphs further show that individual and institutional investor sentiment have some influence on stock market volatility, but the influence is generally weakening, individual investor sentiment has a "self-fulfilling" mechanism and is strengthening, the influence of individual investor sentiment on institutional investor sentiment is increasing.

Keywords

Directed Acyclic Graph; Investor Sentiment; Stock Market Volatility.

1. Introduction

Black [1] (1986) first introduced the concept of "noise" in financial markets, pointing out that noise is content unrelated to company fundamentals, and investors make investment decisions based on such misinformation is called noise trading. The basic framework of noise trading theory was established by De Long et al [2] (1990). As the core theory of behavioral finance, noise trading theory has challenged traditional financial theory. According to traditional finance, markets are efficient and reflect all available information, and even if there is noise trading, the behavior of noise traders can cancel each other out, while rational arbitrageurs in the market can detect the mistakes of noise traders and carry out arbitrage to correct mispricing. And in the long run, noise traders will exit the market due to losses. However, noise trading theory states that the reality of financial markets is "finite arbitrage" and "finite rationality", not only are there many limits to arbitrage behavior, but arbitrageurs may also make different decisions when faced with the same information and exhibit limited rationality. Not only can noise traders exist in the market for a long time, but they can also earn higher returns than rational arbitrageurs due to the additional risk they take. The existence of noise trading in the market has been demonstrated by empirical studies, which suggest that noise

trading can increase the volatility of stock market prices, weaken the effectiveness of the stock market, and reduce stock market liquidity.

In 1990, the Shanghai and Shenzhen Stock Exchanges were established respectively. Over the past 30 years, China's capital market has continued to grow and develop, and by the end of 2022, the market capitalisation of A shares reached RMB87.8 trillion, equivalent to 72.5% of China's annual GDP in 2022, and China's stock market has grown into an important emerging market in the world. While demonstrating the dynamism of an emerging market, China's stock market has also shown characteristics of dramatic volatility.

The irrational investment behavior of market investors is one of the reasons for the dramatic stock market volatility (He Chengying et al. [3], 2021). Compared to the more mature stock markets in Europe and the US, one of the major characteristics of China's stock market is the relatively high level of individual investors and the obvious characteristics of noise trading. The long-standing phenomenon of noise trading in the stock market has led to the prevalence of a speculative culture in the market, making it impossible to give full play to the function of resource allocation in the stock market. Investor sentiment is the behavioral manifestation of noise trading in the market (Yao Yuan et al. [4], 2021). Behavioral finance theory research points out that while stock prices in the market are certainly influenced by fundamental values, changes in investor sentiment can also have a significant impact on investment decision-making behavior, resulting in mispricing in the market. Therefore, it is important to study the impact of investor sentiment on stock market return volatility, so as to guide investors to invest rationally, which is of great practical significance to improve asset pricing efficiency, reduce excessive market volatility and prevent market risks.

2. Literature Review

Behavioral finance argues that investors' emotional investment is an important cause of securities market volatility. Huang Hong [5] (2016) conducted a Granger causality test on the relationship between investor sentiment, financing and financing business, and stock market volatility, the results showed that changes in investor sentiment affect financing and financing business unidirectionally and increase the impact of investor sentiment on stock market volatility. Yao Yuan et al [6] (2019) used the residuals of the regressions of the SSE 50 and SZSI indices as noise, and constructed a noise trading series with the sum of squared residuals, constructed a vector autoregressive model (VAR) model, and used Granger non-causality test, impulse response analysis and variance decomposition to test the relationship between noise trading and stock market volatility. Zhang Bo et al [7] (2021) also constructed a VAR model to empirically test the mechanism of investor sentiment generation.

Previous empirical studies on the interaction between investor sentiment and market volatility have mostly used Granger causality tests or variance decomposition methods. In fact, there are limitations to the Granger causality test, firstly, the Granger causality test is only applicable to determine the temporal sequential relationship, not the factual causality test. Secondly, the results of the Granger causality test are very sensitive to lags.

The variance decomposition based on the VAR model takes into account the significance of the relationship between economic variables in an economic sense and can be used to analyze the interaction between variables, and the correct setting of contemporaneous causality between the disturbance terms is the key to a reasonable variance decomposition. Tong Yuan-song [8] (2021) constructed a structural vector autoregressive (Structural VAR, SVAR) model with three variables: investor sentiment, stock market liquidity, and volatility, and imposed constraints on the SVAR structural equation to identify structural shocks through theoretical analysis and Granger test results. Xie Shiqing and Tang Sixun [9] (2021) similarly identify the SVAR model

based on past literature studies by hypothesizing the relationship between the investor sentiment, macroeconomic volatility, and stock market returns.

Spirtes et al [10] (2000) proposed the "directed acyclic graphs" (DAG) analysis method, which can determine the contemporaneous causality between the perturbations simply by using the residual variance covariance matrix of the data. Yang Zihui[11] (2008) argues that DAG largely overcomes the limitations of traditional analysis methods such as the Granger causality test, and enhances the reliability and rationality of the analysis conclusions. Wang Daoping[12] (2022) used DAG to explore the impact between investor sentiment, excessive trading and stock market volatility in China, and found that investor sentiment in China is an important cause of excessive trading, market volatility also significantly affects investors' trading behavior. Typically, institutional investors have the resource advantage of being able to judge information more accurately in the face of stock market volatility and are able to track the group behavior of individual investors in the market to launch financial investments. Meanwhile, in practice, individual investors cannot be accurately informed of the investment tendencies of institutional investors in a short period of time because the operations of institutional investors are generally disclosed through the published quarterly and annual reports. Based on the above analysis, this paper argues that individual and institutional investor sentiment can have an impact on stock market volatility and that it is mainly individual investor sentiment that affects institutional investor sentiment between individual and institutional investor sentiment. In the following section, investor sentiment indicators are constructed and the forecast error variance decomposition based on DAG analysis is used to investigate the relationship between individual investor sentiment, institutional investor sentiment, and stock market volatility.

3. Model and Data Description

3.1. Model Description

The SVAR model introduces structural relationships between variables of economic and financial theories into the VAR model. First, the VAR models of stock market volatility variables($Volat_t$), individual investor sentiment(S_t^{PI}), and institutional investor sentiment(S_t^{II}) are developed as equation (1).

$$\begin{pmatrix} Volat_t \\ S_t^{PI} \\ S_t^{II} \end{pmatrix} = \sum_{i=1}^p \Gamma_i \begin{pmatrix} Volat_{t-p} \\ S_{t-p}^{PI} \\ S_{t-p}^{II} \end{pmatrix} + u_t \quad (1)$$

In the above equation, Γ_i is the dimensional coefficient matrix, $u_t = (u_{1t}, u_{2t}, u_{3t})^T$ is the VAR reduced-form perturbation term. The above equation is consistent with the AB-SVAR model, and the invertible matrices A and B in the model are set as follows.

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{13} \end{pmatrix} \quad (2)$$

At this point, the AB-SVAR model is set as equation (3).

$$\begin{pmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{pmatrix} = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{13} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \quad (3)$$

In this study, the matrix A represents the current period structural relationship between stock market volatility, individual investor sentiment and institutional investor sentiment variables. u_{1t} is the residual term in period t of the stock market volatility equation, u_{2t} is the residual term in period t of the individual investor sentiment equation, and u_{3t} is the residual term in period t of the institutional investor sentiment equation. matrix B is the diagonal matrix. ε_{1t} , ε_{2t} , and ε_{3t} represent the structural shocks acting on stock market volatility, individual investor sentiment and institutional investor sentiment, respectively.

3.2. Variables and Data Description

3.2.1. Measurement of Stock Market Volatility

The stock market volatility variable is calculated as equation (4).

$$Volat_t = \sqrt{\frac{N}{N_t - 1} \sum_{d=1}^{N_t} \left[(R_{t,d} - \frac{1}{N_t} \sum_{d=1}^{N_t} R_{t,d})^2 \right]} \quad (4)$$

The above equation N refers to the total number of trading days in a month in general, N_t represents the total number of trading days in month t , $R_{t,d}$ represents the CSI 300 index return on the d -th trading day of month t .

3.2.2. Measurement of Investor Sentiment

In this paper, we use two data sources, market trading data and Internet text, to obtain original single indicators that can portray the sentiment of individual and institutional investors. Summarizing the research experience of existing literature and combining the data availability, the market trading indicators are selected as shown in Table 1.

Table 1. Sentiment indicators based on market trading data

Indicator Name	Symbols	Applicable investor types
Number of new individual accounts	I^{NPIA}	Individual investors
Closed-end fund discount rate	I^{DCEF}	Individual investors
Net inflow of small orders	I^{SPA}	Individual investors
Number of new institutional accounts	I^{NIIA}	Institutional investors
Net buying of large orders	I^{LPA}	Institutional investors
Net position of stock index futures	I^{NP}	Institutional investors

The residual series obtained after regressing the selected raw indicators of investor sentiment on the macroeconomic sentiment consensus index is used as the new variable for constructing the composite investor sentiment. The data are normalized in order to eliminate the effect of the magnitude.

At the same time, Senta system was finally selected as the analysis tool in this paper, and the text corpus was selected from the postings of investors in CSI 300 stock bars, and the probability of positive attitude of each stock comment was analyzed as the sentiment tendency score of investors (ranging from 0 to 1). A sentiment score closer to 0 indicates a more pessimistic investor, while a sentiment score closer to 1 indicates a more optimistic investor. The average sentiment score of the month is used as the stock market sentiment indicator of investors according to the time of commenting, and the indicator of individual investors is constructed as I^{TEXT} .

The sample of this paper is selected from August 2015 to April 2022, and the closing price of CSI 300 index is selected to reflect the stock prices. Among the selected market statistical indicators, the data of mega-single and small-single net buying (A-share sectors in Shanghai and Shenzhen) are from Wind database, the rest of market statistical indicators, CSI 300 index and macroeconomic sentiment consensus index data are from CSMAR Guotaian financial research database. The text data come from the Shanghai and Shenzhen 300 posting bar in the Oriental Fortune stock bar, and the crawled content is the title text in the Shanghai and Shenzhen 300 stock bar, and a total of 15,160 text titles were crawled during the sample period.

Referring to Huang [13], the expression of the composite measure of individual investor sentiment index constructed using partial least squares regression method is shown in equation (5).

$$S_t^{PI} = 0.5104I_t^{SPA} + 0.1189I_t^{DCEF} - 0.1437I_t^{NPIA} + 0.3475I_t^{TEXT} \quad (5)$$

The expression of the constructed composite measure of institutional investor sentiment index is shown in equation (6).

$$S_t^{II} = -0.2498I_t^{LPA} + 0.1948I_t^{NNIA} + 0.2913I_t^{NP} \quad (6)$$

4. Analysis of Empirical Results

4.1. SVAR Model Identification

The VAR model requires that the time series used be a stationary time series. ADF test results indicate that stock market volatility, individual sentiment index and institutional investor sentiment index are all stationary series.

The VAR(1) model is developed for $Volat_t$, S_t^{PI} , and S_t^{II} , and the residual correlation matrix is obtained as shown in equation (7).

$$V = \begin{bmatrix} 1 & & \\ 0.1044 & 1 & \\ 0.2265 & 0.6646 & 1 \end{bmatrix} \quad (7)$$

The DAG analysis method was used to apply the TETRAD 7.1 software PC algorithm to analyze the contemporaneous causal relationships between variables based on the residual correlation matrix of equation (7). Since the analysis sample in this paper is 81 observations, the significance level is appropriately relaxed to 20% based on the study of Spirtes et al [10] (2000) on the small sample nature of the DAG method with reference to Yang Zihui [11] (2008). The results of the DAG analysis at the 20% significance level are shown in Figure 1.

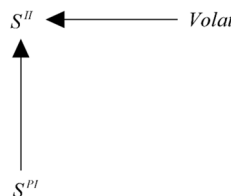


Figure 1. DAG analysis results

The correlation coefficient of S_t^{PI} and $Volat_t$ is 0.1060, and its p-value is 0.3547, which indicates that the unconditional correlation coefficient between these two variables is significantly 0, and there is no contemporaneous causality between S_t^{PI} and $Volat_t$. $Volat_t$ is adjacent to S_t^{II} , and S_t^{PI} is adjacent to S_t^{II} , indicating that there is a contemporaneous causality from stock market

volatility to institutional investor sentiment and from individual investor sentiment to institutional investor sentiment, while there is no significant contemporaneous causality between individual investor sentiment and stock market volatility.

The p-value of the LR statistic is 0.3590, and the original hypothesis that the "overconstraint" is "true" cannot be rejected at the 5% level of significance, which fully justifies the results of the DAG analysis shown in Figure 1.

4.2. Decomposition of Prediction Error Variance

Based on the results of the DAG analysis, the forecast error variance decomposition of the VAR model can further analyze the relationship between market volatility, individual investor sentiment and institutional investor sentiment. Using the sample period August 2015 to April 2022 as the base period, forecasts are made for the next 12 periods, and the results are shown in Tables 2, 3, and 4 below.

The results of the forecast error variance decomposition for stock market volatility in Tables 2 show that the explanatory power of stock market volatility shocks on their own volatility is above 95% over a one-year period. The explanatory power of institutional investor sentiment on stock market volatility variables increases significantly over time, which suggests that shocks to both types of investor sentiment, especially to institutional investor sentiment, can provide some explanation for stock market volatility in the medium to long run.

Table 2. Prediction error variance decomposition of stock market volatility (%)

Forecast period	$Volat_t$	S_t^{PI}	S_t^{II}
1	100.0000	0.0000	0.0000
3	98.2006	0.5591	1.2403
6	96.7948	0.6339	2.5713
9	96.3774	0.6651	2.9576
12	96.2436	0.6968	3.0596

The results of the forecast error variance decomposition for individual investor sentiment in Tables 3 show that in forecast period 1, the individual investor sentiment variable is fully explained by itself, which is consistent with the results of the DAG analysis. As the forecast period is extended, the percentage of individual investor sentiment fluctuations explained by own perturbations is consistently above 98%. In the medium and long term, institutional investor sentiment increases in explaining individual investor sentiment, but stock market volatility is consistently less explained by individual investor sentiment. This suggests that the fluctuations in individual investor sentiment mainly come from inertial movements due to changes in their own perceptions.

Table 3. Prediction error variance decomposition of individual investor sentiment (%)

Forecast period	$Volat_t$	S_t^{PI}	S_t^{II}
1	0.0000	100.0000	0.0000
3	0.1056	99.6335	0.2609
6	0.2003	99.0455	0.7543
9	0.2250	98.7855	0.9894
12	0.2312	98.6999	1.0689

The results of the forecast error variance decomposition for institutional investor sentiment in Tables 4 show that in the first period of the forecast, about 43% of the fluctuations in institutional investor sentiment are caused by shocks to individual investor sentiment and about 2.6% by shocks to stock market volatility variables. In the medium to long term, this result does not change much. This result suggests that changes in individual investor sentiment drive changes in institutional investor sentiment, while stock market volatility has less explanatory power for institutional investor sentiment.

Table 4. Prediction error variance decomposition of institutional investor sentiment (%)

Forecast period	$Volat_t$	S_t^{PI}	S_t^{II}
1	2.5786	42.9136	54.5078
3	1.4113	44.5005	54.0882
6	1.5932	44.0881	54.3187
9	1.7051	43.7598	54.5351
12	1.7389	43.6421	54.6191

5. Conclusion

This paper uses DAG analysis to determine the contemporaneous causality between individual investors, institutional investors, and stock market volatility. The empirical results show that in terms of contemporaneous interactions, there is contemporaneous causality from stock market volatility to institutional investor sentiment and from individual investor sentiment to institutional investor sentiment. The results of the prediction error variance decomposition for stock market volatility indicate that stock market volatility shocks explain their own volatility more strongly in the short run, and the explanatory power of individual and institutional investor sentiment to stock market volatility increases in the medium and long run. The results of the prediction error variance decomposition for individual investor sentiment suggest that individual investor sentiment has a strong "self-fulfilling" mechanism. The forecast error variance decomposition results for institutional investor sentiment suggest that individual investor sentiment is always an important influence on institutional investor sentiment in both the short and medium term.

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